Academic Journal of Science,
CD-ROM. ISSN: 2165-6282 :: 04(01):125–135 (2015)

SYNCHRONIZATION BETWEEN CHUA AND MODIFIED CHUA OSCILLATORS WITH PASSIVE CONTROL

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In this study, the synchronization between Chua and modified Chua oscillators is investigated by means of the passive control method. First, the Chua and cubic Chua oscillators are described and defined as a set of differential equations. Their chaotic time series and phase portraits are demonstrated. Then, the passive controllers are constructed for the synchronization. The global asymptotical stability of errors between the Chua oscillators is ensured with the Lyapunov function. Afterwards, numerical results are demonstrated to confirm the theoretical analysis. They have also shown that the passive controllers are effectively used for synchronizing two different Chua oscillators.

Keywords: Chua's circuit, Modified Chua oscillator, Double scroll, Chaos synchronization, Passive control.

Introduction

Since Lorenz introduced the first chaotic attractor in 1963 [1], the investigations on chaos have become an important subject in the engineering area and many chaotic attractors have been discovered. A new three-dimensional chaotic Rössler system was proposed in 1976 [2]. Chua's double-scroll attractor was presented in 1984 [3]. Sprott focused on simpler chaotic systems in 1994 and uncovered 19 distinct chaotic flows which have either five terms and two nonlinearities or six terms and one nonlinearity [4]. Chen attractor was proposed by Chen and Ueta in 1999 [5]. Lü et al. discovered a new chaotic system, which represents the transition between the Lorenz and Chen systems, in 2002 [6]. Then, Lü et al. suggested a generalized form of the Lorenz, Chen and Lü systems called unified chaotic system [7]. In recent years, several new chaotic attractors have also been proposed [8–10].

Besides new chaotic attractors, the synchronization of chaotic systems becomes an important task and has received increasing attention from researchers due to their potential applications especially in secure communication [11, 12]. The idea of synchronizing two identical chaotic systems was introduced by Pecora and Carroll in 1990 [13]. Then, several methods have been successfully applied to synchronize the chaotic systems such as active control [14], passive control [15–19], sliding mode control [20], impulsive control [21], backstepping design [22] and so on. Among them, passive control is a significant chaos control method. It has been used for the synchronization of Chen [15], unified [16], Rikitake [17], between Rössler and Genesio-Tesi [18], and many other chaotic systems [19].

Introduced by Leon O. Chua, Chua's circuit is a simple electronic circuit that consists of one linear resistor, two capacitors, one inductor, and one nonlinear resistor. Its simplicity and chaotic phenomena make the Chua oscillator a well-known circuit. Its dynamical behaviours and properties have been extensively investigated in many papers [23–25]. Some modified versions of Chua's circuits are also proposed [26, 27]. The control and synchronization of Chua systems have received lots of attentions from researchers. Recently, the synchronization of Chua oscillators is implemented with active control [28], adaptive control [29], sliding mode control [30], fuzzy control [31], impulsive control [32], and backstepping design [33] methods. According to the literature review, there is no passive control approach for the synchronization of Chua oscillators. In this paper, the synchronization between two different chaotic Chua oscillators is applied with the passive control method.

System Descriptions

As seen in Fig.1, the Chua's circuit contains 5 circuit elements; inductance L, resistance R, two capacitances C_1 and C_2, and three-segment nonlinear resistor g [3].

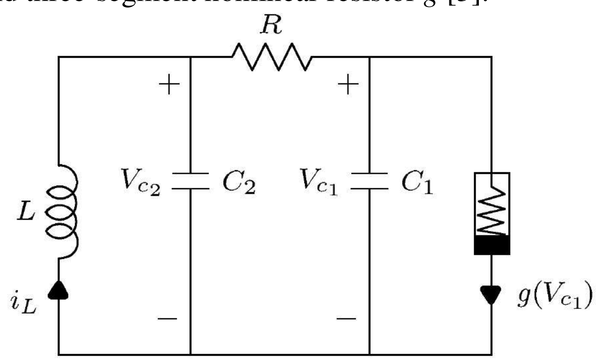


Fig. 1. The Chua's circuit.

The Chua's system is described by the following dimensionless form

$$\begin{aligned} \dot{x} &= \alpha(y - x - f(x)), \\ \dot{y} &= x - y + z, \\ \dot{z} &= -\beta y, \end{aligned} \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are the system parameters determined by the particular values of the circuit components; $f(x)$ is the function describes the electrical response of the nonlinear resistor; x, y, and z are the state variables represent the voltages across the capacitors C_1 and C_2, and the intensity of the electrical current in the inductor L, respectively. The $f(x)$ function depends on the particular configuration of components. It is generally considered as

$$f(x) = -bx - \frac{1}{2}(a-b)(|x+1| - |x-1|). \tag{2}$$

Chua's circuit exhibits chaotic phenomena when the parameters are selected as $\alpha = 9$, $\beta = 100/7$, $a = 8/7$, and $b = 5/7$ with the initial condition (0, 0, 0.6) [3]. The time series of Chua oscillator are demonstrated in Fig. 2, the 2D phas
portraits are demonstrated in Fig. 3, and the 3D phase plane is demonstrated in Fig. 4. Chua oscillator is also known as "double scroll" because of its shape in the (x, y, z) space.

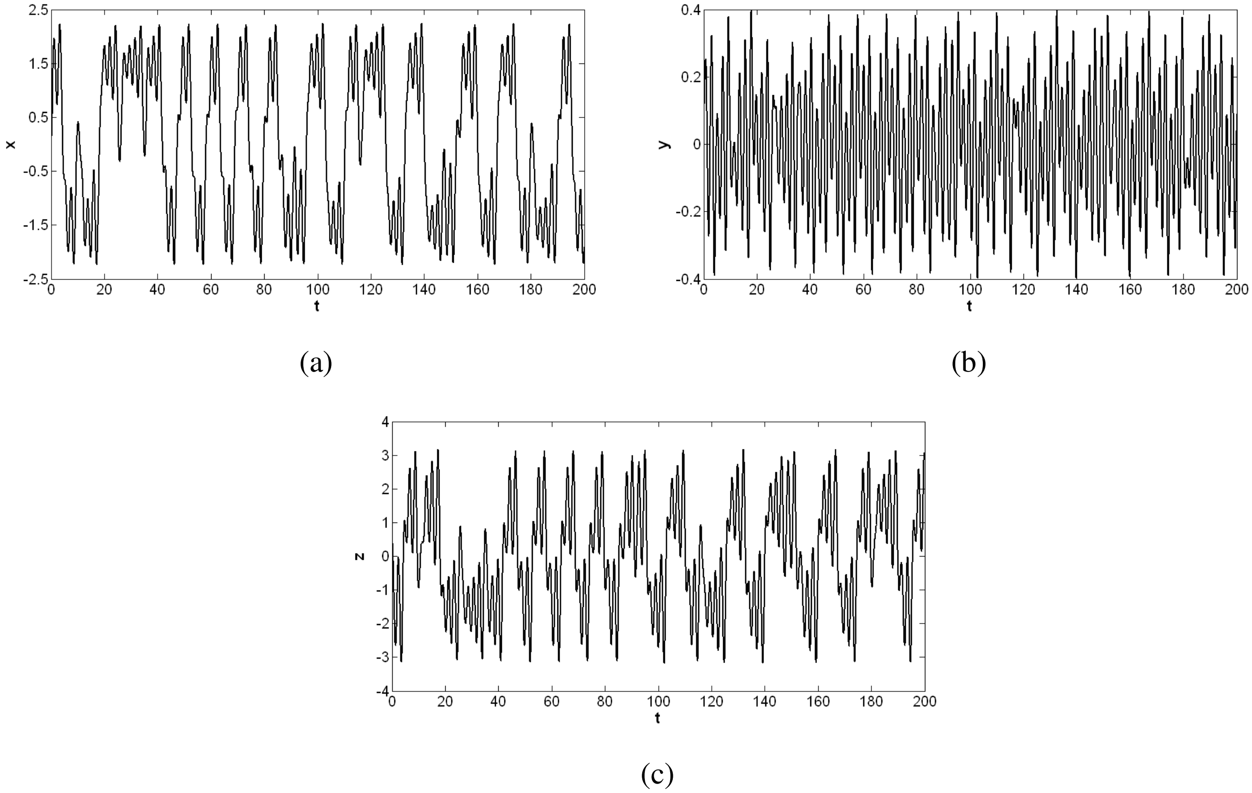
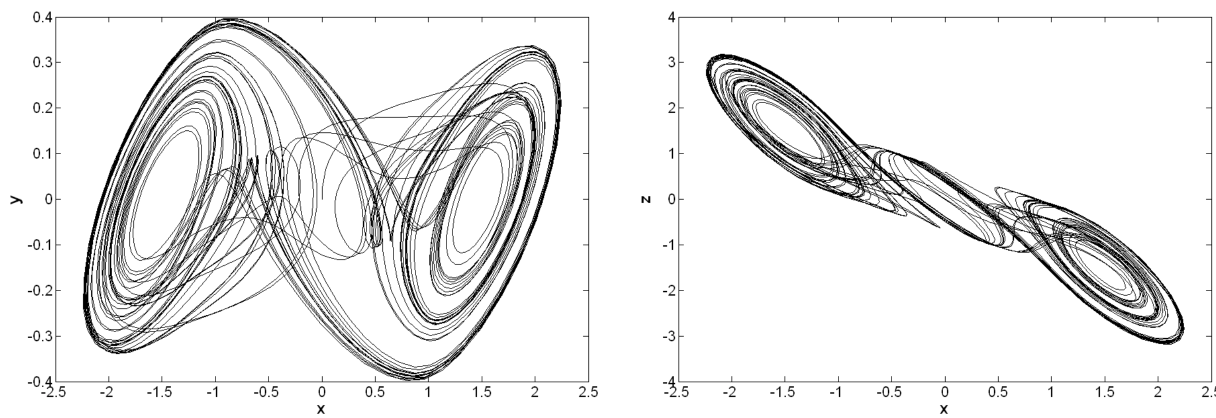
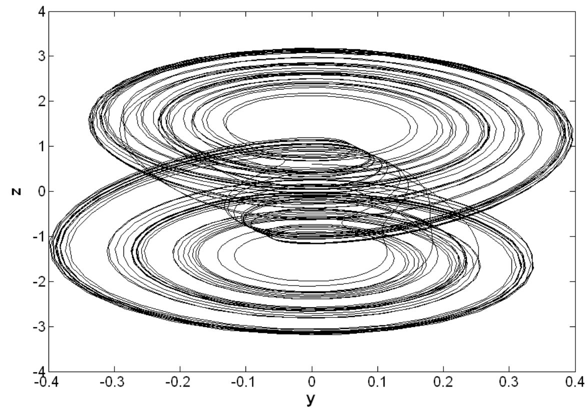


Fig. 2. Time series of Chua oscillator for (a) *x* signals, (b) *y* signals, (c) *z* signals.



(a) (b)



(c)

Fig. 3. 2D phase portraits of Chua oscillator for (a) *x*–*y* phase plot, (b) *x*–*z* phase plot, (c) *y*–*z* phase plot.

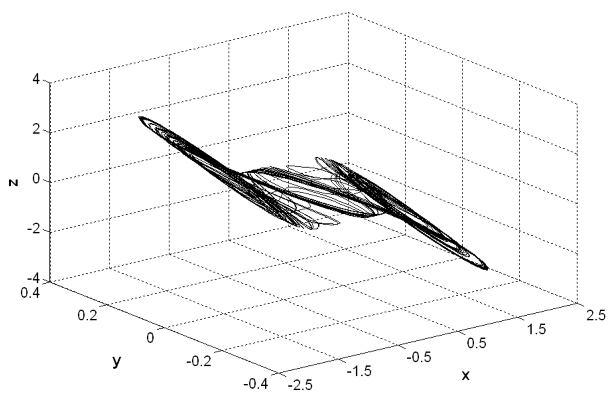


Fig. 4. 3D phase plane of Chua oscillator.

Hartly proposed a modified Chua oscillator in 1989 [26]. A cubic nonlinearity is used in place of the piecewise-linear nonlinearity of Chua's circuit. It changes the dynamics of system and the bifurcation structure a little. The cubic Chua system is described by

$$\begin{aligned} \dot{x} &= \alpha(y - \frac{1}{7}(2x^3 - x)), \\ \dot{y} &= x - y + z, \\ \dot{z} &= -\beta y, \end{aligned} \tag{3}$$

where $\alpha > 0$ and $\beta > 0$ are the system parameters; x and y are the voltages across the two capacitors; and z is the current through the inductor. The cubic Chua's circuit system displays chaotic behaviour when the parameters are considered as $\alpha = 10$ and $\beta = 100/7$ with the initial condition (0.3, 0, 0) [34]. The time series, the 2D phase portraits, and the 3D phase plane of cubic Chua oscillator are demonstrated in Fig. 5, Fig. 6, and Fig. 7, respectively.

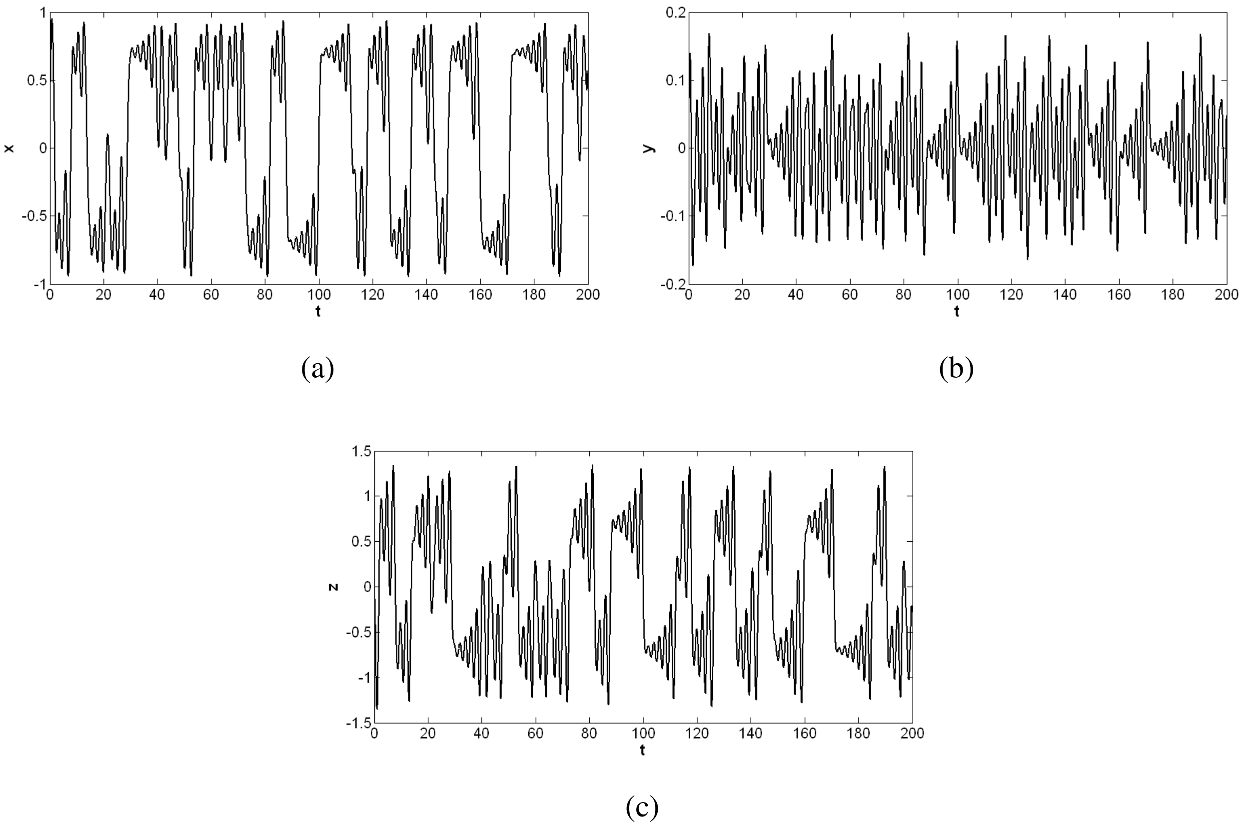
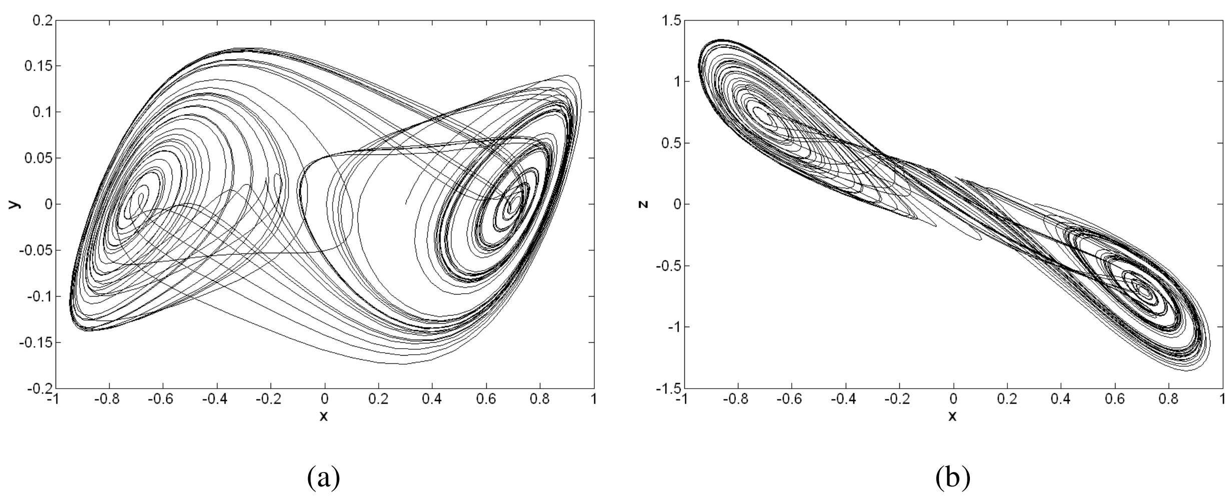
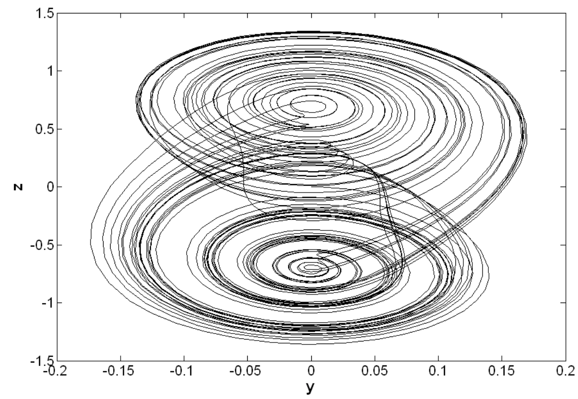


Fig. 5. Time series of cubic Chua oscillator for (a) *x* signals, (b) *y* signals, (c) *z* signals.





(c)

Fig. 6. 2D phase portraits of cubic Chua oscillator for (a) *x*–*y* phase plot, (b) *x*–*z* phase plot, (c) *y*–*z* phase plot.

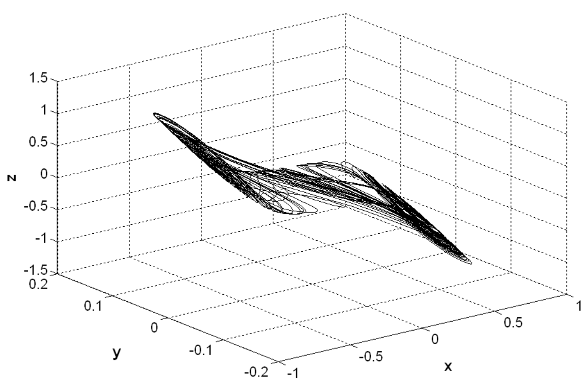


Fig. 7. 3D phase plane of cubic Chua oscillator.

Synchronization

In this section, the synchronization of two different Chua oscillators with unknown parameters is applied by using a passivity-based control method. Two chaotic Chua systems are taken where the drive Chua oscillator denoted by the subscript 1 controls the response cubic Chua oscillator which is denoted by subscript 2. The drive and response Chua systems are defined as follows:

$$\begin{aligned} \dot{x}_1 &= \alpha_1(y_1 - x_1 - f(x_1)), \\ \dot{y}_1 &= x_1 - y_1 + z_1, \\ \dot{z}_1 &= -\beta_1 y_1, \end{aligned} \tag{4}$$

and

$$\begin{aligned} \dot{x}_2 &= \alpha_2(y_2 - \frac{1}{7}(2x_2^{\ 3} - x_2)) + u_1, \\ \dot{y}_2 &= x_2 - y_2 + z_2, \\ \dot{z} &= -\beta_2 y_2 + u_2, \end{aligned} \tag{5}$$

where u_1 and u_2 in Equation (5) are the control functions to be determined. In order to obtain the control functions for synchronization, the drive system is subtracted from the response system. The e_1, e_2, and e_3 state errors between cubic Chua chaotic system (5) that is to be controlled and the controlling Chua chaotic system (4) is defined as

$$\begin{aligned} e_1 &= x_2 - x_1, \\ e_2 &= y_2 - y_1, \\ e_3 &= z_2 - z_1. \end{aligned} \tag{6}$$

It leads to

$$\begin{aligned} \dot{e}_1 &= -\alpha_1(y_1 - x_1 - f(x_1)) + \alpha_2(y_2 - \frac{1}{7}(2x_2^{\ 3} - x_2)) + u_1, \\ \dot{e}_2 &= e_1 - e_2 + e_3, \\ \dot{e}_3 &= \beta_1 y_1 - \beta_2 y_2 + u_2. \end{aligned} \tag{7}$$

The system (7) is called the error system. The synchronization problem is to ensure the error system (7) asymptotically stable at the zero equilibrium point. By assuming that the state variable e_1 is the output of the system and supposing $Z_1 = e_2$, $Z_2 = e_3$, $Y = e_1$, $Z = [Z_1\ Z_2]^T$, then system (7) can be denoted by normal form:

$$\begin{aligned} \dot{Z}_1 &= -Z_1 + Z_2 + Y, \\ \dot{Z}_2 &= \beta_1 y_1 - \beta_2 y_2 + u_2, \\ \dot{Y} &= -\alpha_1(y_1 - x_1 - f(x_1)) + \alpha_2(y_2 - \frac{1}{7}(2x_2^{\ 3} - x_2)) + u_1. \end{aligned} \tag{8}$$

The passive control theory has the following generalized form:

$$\begin{aligned} \dot{Z} &= f_0(X) + p(Z,Y)Y, \\ \dot{Y} &= b(Z,Y) + a(Z,Y)u, \end{aligned} \tag{9}$$

and according to system (8):

$$\begin{aligned} f_0(Z) &= \begin{bmatrix} -Z_1 + Z_2 \\ \beta_1 y_1 - \beta_2 y_2 + u_2 \end{bmatrix}, \\ p(Z,Y) &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ b(Z,Y) &= -\alpha_1(y_1 - x_1 - f(x_1)) + \alpha_2(y_2 - \frac{1}{7}(2x_2^{\ 3} - x_2)), \\ a(Z,Y) &= 1. \end{aligned} \tag{10}$$

The storage function is chosen as

$$V(Z,Y) = W(Z) + \frac{1}{2}(Y^2), \tag{11}$$

where $W(Z) = \frac{1}{2}(Z_1^2 + Z_2^2)$ is the Lyapunov function of $f_0(Z)$ with $W(0) = 0$.

In order to obtain derivative of $W(Z)$ as negative, the control function u_2 is considered by:

$$u_2 = -\beta_1 y_1 + \beta_2 y_2 - Z_1. \tag{12}$$

According to Equation (10), by taking the derivative of $W(Z)$

$$\begin{aligned}\dot{W} &= \frac{d}{dt}W(Z) = \frac{\partial W(Z)}{\partial Z} f_0(Z) = \begin{bmatrix} Z_1 & Z_2 \end{bmatrix} \begin{bmatrix} -Z_1 + Z_2 \\ -Z_1 \end{bmatrix} \\ &= -Z_1^{\ 2} + Z_1 Z_2 - Z_1 Z_2 = -Z_1^{\ 2}.\end{aligned} \tag{13}$$

Since $W(Z) \geq 0$ and $\dot{W}(Z) \leq 0$, it can be concluded that $W(Z)$ is the Lyapunov function of $f_0(Z)$ and that $f_0(Z)$ is globally asymptotically stable [17]. So, the zero dynamics of the error system (8) is stable based on the Lyapunov stability. The synchronized system can be equivalent to a passive system and globally asymptotically stabilized at its zero equilibrium by the following state feedback controller [16]:

$$u = a(Z,Y)^{-1}\left[-b^T(Z,Y) - \frac{\partial W(Z)}{\partial Z} p(Z,Y) - \alpha Y + v \right]. \tag{14}$$

Accordingly, it is determined for the error system (8) as

$$u_1 = \alpha_1(y_1 - x_1 - f(x_1)) - \alpha_2(y_2 - \frac{1}{7}(2x_2^{\ 3} - x_2)) - Z_1 - \alpha Y + v, \tag{15}$$

where α is a positive constant and v is an external input signal. By taking back $Z_1 = e_2$, $Z_2 = e_3$, $Y = e_1$ conversions, the control functions become

$$\begin{aligned} u_1 &= \alpha_1(y_1 - x_1 - f(x_1)) - \alpha_2(y_2 - \frac{1}{7}(2x_2^{\ 3} - x_2)) - e_2 - \alpha e_1 + v, \\ u_2 &= -\beta_1 y_1 + \beta_2 y_2 - e_2. \end{aligned} \tag{16}$$

Hence, the synchronization between Chua and cubic Chua oscillators with unknown parameters is achieved by means of passive control method.

Numerical Simulations

In this section, computer simulations are performed to show the synchronization between Chua oscillator and cubic Chua oscillator with the passive control method. The numerical analysis is carried out using Runge-Kutta method of order 4 with variable time step. The parameters of Chua's circuit are taken as α_1 = 9, β_1 = 100/7, a = 8/7, and b = 5/7; the initial condition is (0, 0, 0.6). The parameters of cubic Chua oscillator are taken as α_2 = 10 and β_2 = 100/7; the initial condition is (0.3, 0, 0). The passive controller parameters are chosen as α = 10 and v = 0. In order to present the chaotic trajectories before and after the control law is applied, the controllers are activated at t = 25. The simulation results of synchronization are demonstrated in Fig. 8, and the error signals are demonstrated in Fig. 9.

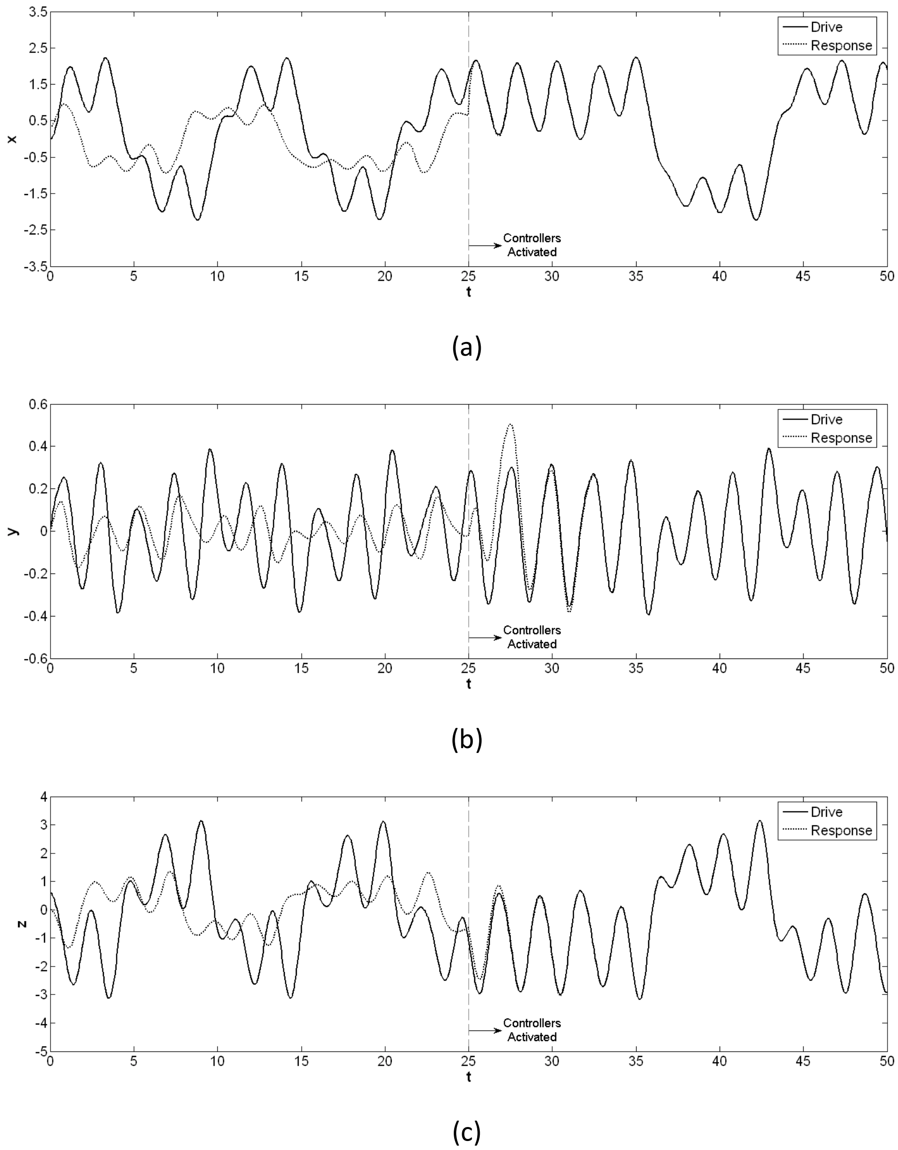


Fig. 8. The time response of states for the synchronization between chaotic Chua and cubic Chua oscillators when the passive controllers are activated at $t = 25$ for (a) x signals, (b) y signals, (c) z signals.

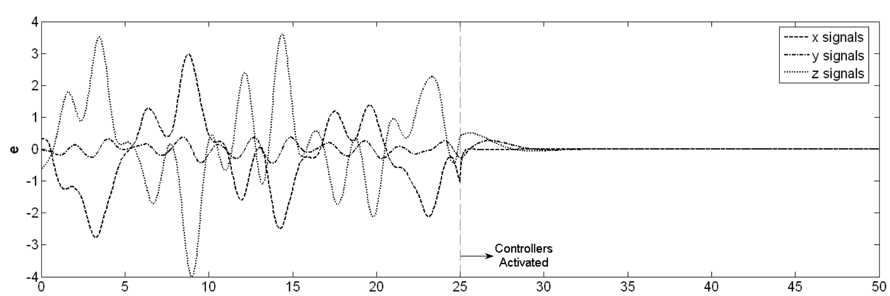


Fig. 9. The time response of error signals for the synchronization between chaotic Chua and cubic Chua oscillators when the passive controllers are activated at $t = 25$.

As expected, Fig. 8 from Matlab–Simulink outputs shows that the passive controllers have achieved the non-identical synchronization of chaotic Chua oscillators. The error signals that are shown in Fig. 9 converge asymptotically to zero. Therefore, all the analytical results have been confirmed by the computer simulations. When the controllers are activated at $t = 25$, the synchronization is obtained at $t \geq 32$ with the passive controllers. Hence, the passive controllers are appropriate for the synchronization of two different Chua oscillators.

Conclusion

In this paper, the passive control method is proposed for the synchronization between chaotic Chua and cubic Chua oscillators with unknown parameters. Based on the properties of passivity, the passive controllers have been constructed to realize the synchronization. Simulation results show that the passive controllers are able to synchronize the chaotic motion of cubic Chua oscillator to the chaotic motion of Chua's circuit. So, the theoretical analyses are confirmed. Numerical simulations also show that the proposed passive control method is effective for the non-identical chaos synchronization of Chua oscillators.

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