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THE ECONOMIC AND ECONOMIC-STATISTICAL DESIGNS OF THE COEFFICIENT OF VARIATION CHART

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Superior quality of manufactured products ensures sustainability in industries. However, the cost of monitoring and improving quality can be high. To overcome the high cost of quality control, this paper proposes economic and economic-statistical designs of the coefficient of variation (CV) chart. The CV chart monitors the ratio of the standard deviation to the mean, and is useful in processes where the mean is not constant and/or the variance is a function of the mean. In these processes, the traditional $\bar{X}$ and *S* or *R* charts cannot be used. Although many studies are done on the CV chart, an economic model which minimizes the cost of implementing the CV chart cannot be found in the existing literature. Thus, this paper proposes a simplified algorithm to obtain the optimal chart parameters of the CV chart, i.e. the sample size, the sampling interval, the upper control limit and the lower control limit, which minimize the expected cost function. Two models are considered. In the economic design, the expected cost function is minimized without the need to satisfy statistical constraints; while for the economic-statistical design, the expected cost function is minimized subject to statistical constraints. A sensitivity analysis to identify the input parameters which have a significant impact on the cost and choice of optimal chart's parameters of the CV chart are performed based on numerical examples. Besides that, the effects of adding statistical constraints are investigated.

Keywords: Coefficient of variation, Economic design, Economic-statistical design, Sensitivity analysis, Statistical constraint.

Introduction

Control charts are one of the important tools adopted to monitor processes. Control charts allow the practitioner to statistically control and monitor one or more variables, by determining whether the process or quality characteristics is "in-control" or "out-of-control". An "in-control" process operates with only chance causes of variation, whereas an "out-of-control" process operates in the presence of assignable causes. For an out-of-control process, actions are taken to find and eliminate the assignable cause or causes before a large quantity of defective products are produced. A good control chart can distinguish between chance causes of variation, or "background noise", and abnormal variation, which prevent unnecessary process adjustment.

In the existing literature, most of the control charts monitor changes in the mean (μ) and/or standard deviation (σ). Thus, a shift in the mean and/or standard deviation makes the process out-of-control. However, there are many in-control processes where the mean is expected to fluctuate and the standard deviation changes with the mean. This frequently happens when we monitor some process outputs which typically change from time to time. For example, the amount of a certain chemical in a patient's blood varies from patient to patient. Another example is monitoring the contaminants released day to day in the atmosphere by industrial plants, where the amount of processing can change based on a daily schedule. Besides that, there are many situations in which consumers are more concerned about the stability of product quality. For example, the mean of each batch of steel may be different because of different techniques and different proportion of ingredients or other factors in a steel factory. Usually steel users are more concerned about just what batch of steel of which tensile strength is more stable (that is less variability), but do not care about the average tensile strength. This is because the stability of tensile strength is very important for the enterprises that use the steel in batches to complete remanufacturing production (Wang and Xiao, 2001). In such processes, the use of the traditional $\bar{X}$ and S or R charts is dubious. However, if the process standard deviation is a linear function of the process mean, the sample coefficient of variation (CV) can be used to keep track of the process variability and to detect shifts in the process mean or standard deviation due to assignable causes.

Kang *et al.* (2007) was the first to propose a control chart to monitor the CV. In this chart, the process is considered to be in-control as long as the CV, i.e. $\gamma = \dfrac{\sigma}{\mu}$ is constant, even though there may be changes in μ and/or σ. The process is declared as out-of-control when an assignable cause changes the relation between μ and σ.

The CV chart proposed by Kang *et al.* (2007) is a Shewhart-type chart, thus it is only sensitive to large shifts in the CV. Therefore, several studies are done to improve the ability to detect small and moderate shifts in the CV. Hong *et al.* (2008) and Castagliola *et al.* (2011) proposed an Exponentially Weighted Moving Average (EWMA) chart to monitor the CV; while Calzada and Scariano (2013) proposed a synthetic chart to monitor the CV. Next, Castagliola *et al.* (2013a) proposed a CV chart using runs rule. Recently, Castagliola *et al.* (2013b) and Castagliola *et al.* (2015) monitored the CV using variable sampling interval and variable sample size charts, respectively; while Zhang *et al.* (2014) proposed a modified EWMA chart to improve the sensitivity of the EWMA CV chart. Finally, Yeong *et al.* (2015a) proposed the multivariate version of the CV chart.

In today's competitive business environment, minimizing cost is very important. Cost optimization models for control charts are vital to reduce the cost of quality control without sacrificing its effectiveness. The economic design of control charts was first proposed by Duncan (1956), and later generalized by Lorenzen and Vance (1986). However, it was found that the economic design results in poor statistical properties, mainly in terms of a high false alarm rate. Thus, Saniga (1989) proposed the economic-statistical design, where statistical constraints are added. Some of the recent studies in economic and economic-statistical designs are Yeong *et al.* (2013), Yilmaz and Burnak (2013), Yeong *et al.* (2014), Niaki *et al.* (2014), Noorossana *et al.* (2014), Amiri *et al.* (2015), Chiu (2015) and Yeong *et al.* (2015b).

However, the cost optimization model for the CV chart cannot be found in the existing literature. This research will fill this gap by proposing economic and economic-statistical designs for the CV chart. Through the economic design, practitioners will be able to choose the chart parameters of the CV chart in order to minimize the cost. The chart parameters of the CV chart refer to the sample size, the sampling interval and the control limit coefficient. The economic-statistical design, i.e. the economic design with statistical constraints, will also be proposed for the CV chart, so that the statistical performance of the chart is not sacrificed. In this paper, we set the constraints in terms of the in-control and out-of-control average run length (ARL_0 and ARL_1).

This paper is organized as follows. In the next section, an overview of the CV chart is provided. This is followed by the economic and economic-statistical designs of the CV chart. Subsequently, these designs are implemented on numerical examples. Through the numerical examples, a sensitivity analysis is performed to identify the input parameters which have a significant effect on the cost and choice of chart's parameters. The effects of adding statistical constraints are also investigated. Finally, some concluding remarks are given.

Coefficient of Variation (CV) Chart

In this section, a brief overview of the CV chart proposed by Kang *et al.* (2007) is provided. Kang *et al.* (2007) proposed a Shewhart-type chart to monitor the CV. The chart gives an out-of-control signal when the sample coefficient of variation $\hat{\gamma}$ falls outside the upper and lower control limits. To compute the sample coefficient of variation, a sample of *n* independent and identically distributed normal random variable $\{X_1, X_2, ..., X_n\}$ are taken, where $X_i \sim N\left(\mu, \sigma^2\right), i = 1, 2, ..., n$. Let $\bar{X}$ and S be the sample mean and sample standard deviation, respectively, where

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{1}$$

and

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}. \tag{2}$$

$\hat{\gamma}$ is defined as

$$\hat{\gamma} = \frac{S_i}{\bar{Y}_i}. \tag{3}$$

Iglewicz *et al.* (1968) noticed that $\frac{\sqrt{n}}{\hat{\gamma}}$ follows a non-central *t*-distribution with *n*-1 degrees of freedom and non-centrality parameter $\frac{\sqrt{n}}{\gamma}$. Thus, from Castagliola *et al.* (2011), the c.d.f. (cumulative distribution function) of $\hat{\gamma}$ is as follows:

$$F_{\hat{\gamma}}(y \mid n, \gamma) = 1 - F_t\left(\frac{\sqrt{n}}{y} \mid n-1, \frac{\sqrt{n}}{\gamma}\right) \tag{4}$$

where $F_t(.)$ is the c.d.f of the non-central *t*-distribution with *n*-1 degrees of freedom and non centrality parameter $\frac{\sqrt{n}}{\gamma}$. Inverting $F_{\hat{\gamma}}(y \mid n, \gamma)$ gives the inverse c.d.f. $F_{\hat{\gamma}}^{-1}(y \mid n, \gamma)$ of $\hat{\gamma}$ as

$$F_{\hat{\gamma}}^{-1}(y \mid n, \gamma) = \frac{\sqrt{n}}{F_t^{-1}\left[1 - \alpha \mid n-1, \frac{\sqrt{n}}{\gamma}\right]}, \tag{5}$$

where $F_{\hat{\gamma}}^{-1}(.)$ is the inverse c.d.f. of the non-central *t*-distribution.

The lower and upper control limits (LCL and UCL) can be computed as follows:

$$LCL = \mu_0(\hat{\gamma}) - k\sigma_0(\hat{\gamma}) \tag{6}$$

and

$$UCL = \mu_0(\hat{\gamma}) + k\sigma_0(\hat{\gamma}) \tag{7}$$

where *k* is the control limit coefficient which affects the width of the in-control region, and is determined by the practitioner.

In Equations (6) and (7), $\mu_0(\hat{\gamma})$ and $\sigma_0(\hat{\gamma})$ are the mean and standard deviation of $\hat{\gamma}$ when the process is in-control, i.e. $\gamma_i = \gamma_0$. Since there is no closed form for $\mu_0(\hat{\gamma})$ and $\sigma_0(\hat{\gamma})$, the approximations proposed by Reh and Scheffler (1996) are adopted in this paper, i.e.

$$\mu_0(\hat{\gamma}) \approx \gamma_0\left[1 + \frac{1}{n}\left(\gamma_0^2 - \frac{1}{4}\right) + \frac{1}{n^2}\left(3\gamma_0^4 - \frac{\gamma_0^2}{4} - \frac{7}{32}\right) + \frac{1}{n^3}\left(15\gamma_0^6 - \frac{3\gamma_0^4}{4} - \frac{7\gamma_0^2}{32} - \frac{19}{28}\right)\right], \tag{8}$$

and

$$\sigma_0(\hat{\gamma}) \approx \gamma_0\sqrt{\frac{1}{n}\left(\gamma_0^2 + \frac{1}{2}\right) + \frac{1}{n^2}\left(8\gamma_0^4 + \gamma_0^2 + \frac{3}{8}\right) + \frac{1}{n^3}\left(69\gamma_0^6 + \frac{7\gamma_0^4}{2} + \frac{3\gamma_0^2}{4} + \frac{3}{16}\right)}. \tag{9}$$

The out-of-control average run length (ARL_1) for different shifts $\tau = \frac{\gamma_1}{\gamma_0}$, where γ_0, γ_1 and τ are the in-control CV, out-of-control CV and size of shift, respectively, can be computed by using the c.d.f. of $\hat{\gamma}$ in Equation (4). By letting

$$\begin{aligned} P &= P(\hat{\gamma} < LCL) + P(\hat{\gamma} > UCL) \\ &= 1 + F_{\hat{\gamma}}(LCL \mid n, \gamma_1) - F_{\hat{\gamma}}(UCL \mid n, \gamma_1), \end{aligned} \tag{10}$$

where $F_{\hat{\gamma}}(.)$ is defined in Equation (4), the ARL_1 is then computed as

$$\text{ARL}_1 = \frac{1}{P}. \tag{11}$$

The in-control average run length (ARL_0) can be obtained from Equation (11) by letting $\tau = 1$.

Economic and Economic-statistical Designs of the CV Chart

The general cost function of Lorenzen and Vance (1986) is modified and used to determine the optimal values of the CV chart's parameters. The process is assumed to follow a normal distribution with mean μ and standard deviation σ. When $\frac{\sigma}{\mu} = \gamma_0$, the process is in-control. It is assumed that the process starts at an in-control condition. There will be a shift in the process coefficient of variation when an assignable cause occurs, so that the new coefficient of variation is $\gamma_1 = \tau\gamma_0$. The assignable cause results in an out-of-control process. The length of time until the assignable cause occurs is exponentially distributed random variable with rate λ. Once an assignable cause is detected, actions are taken to find and eliminate the assignable cause to bring the process back to an in-control condition $\left(\dfrac{\sigma}{\mu}=\gamma_0\right)$. Note that the process is considered to be in-control even though there are changes in the means and standard deviation, as long as the ratio $\dfrac{\sigma}{\mu}$ remains constant.

The total cost includes all the cost in the in-control and out-of-control states, the cost of sampling, the cost of false alarms and the cost of repairs. The expected cost per unit time, C is obtained as follows:

$$C=\frac{\dfrac{C_0}{\lambda}+C_1B+\dfrac{b+cn}{h}(\dfrac{1}{\lambda}+B)+\dfrac{sY}{\mathrm{ARL}_0}+W}{\dfrac{1}{\lambda}+\dfrac{(1-\varphi_1)sT_0}{\mathrm{ARL}_0}+EH} \tag{12}$$

where,

$B=(\mathrm{ARL}_1-0.5)h+F$,

$F=ne+\varphi_1T_1+\varphi_2T_2$,

$EH=(\mathrm{ARL}_1-0.5)h+G$,

$G=ne+T_1+T_2$,

and

$s=\dfrac{1}{\lambda h}-\dfrac{1}{2}$.

The expected cost per unit time is obtained by dividing the expected cost per cycle with the expected cycle time, where each cycle is defined as the start of successive in-control periods. The derivation of the cost function is shown in Lorenzen and Vance (1986). The parameters in Equation (12) are defined as follows:

ARL_0	Average run length while in control
ARL_1	Average run length while out-of-control
b	Fixed cost per sample
c	Cost per unit sampled
C	Expected cost per hour
C_0	Expected quality cost per hour while in control
C_1	Expected quality cost per hour while out-of-control
e	Expected time to sample and interpret one unit
h	Sampling interval
n	Sample size
s	Expected number of samples taken before an assignable cause occur
T_0	Expected search time for a false alarm
T_1	Expected time to find the assignable cause
T_2	Expected time to repair the process

W	Cost of finding and fixing an assignable cause
Y	Cost of false alarm
φ_1	=1 if production continues during search =0 if production stops during search
φ_2	=1 if production continues during repair =0 if production stops during repair
λ	Expected failure time

The optimal design parameters of the CV chart are the chart parameters of the CV chart that minimize C in Equation (12). In other words, it is the values of the sample size (n), control limit coefficient for the CV chart (k), and the sampling interval (h) which minimizes C.

The optimal h is obtained from the following equation:

$$h = \frac{-r_2 + \sqrt{r_2^2 - 4r_1r_3}}{2r_1}, \tag{13}$$

where

$$r_1 = \frac{(\mathrm{ARL}_1 - 0.5)\left(\lambda(Y + C_1T_0(-1+\varphi_1)) - 2\mathrm{ARL}_0\left(C_0 + \lambda((\mathrm{ARL}_1 - 0.5)b + (\mathrm{ARL}_1 - 0.5)cn + W) + C_1(-1 + F\lambda - G\lambda)\right)\right)}{2\lambda\mathrm{ARL}_0},$$

$$r_2 = -\frac{2(\mathrm{ARL}_1 - 0.5)\left(Y + C_1T_0\left(-1+\varphi_1\right) + \mathrm{ARL}_0\left(b+cn\right)\left(1+F\lambda\right)\right)}{\lambda\mathrm{ARL}_0},$$

and

$$r_3 = -\frac{1}{2\lambda^2\mathrm{ARL}_0}\begin{bmatrix} 2Y + 2C_0T_0\left(-1+\varphi_1\right) - bT_0\lambda - 2\left(\mathrm{ARL}_1 - 0.5\right)bT_0\lambda - 2C_1FT_o\lambda - cnT_0\lambda \\ -2\left(\mathrm{ARL}_1 - 0.5\right)cnT_0\lambda - 2T_0W\lambda + 2GY\lambda + bT_0\varphi_1\lambda + 2\left(\mathrm{ARL}_1 - 0.5\right)bT_0\varphi_1\lambda \\ +2C_1FT_0\varphi_1\lambda + cnT_0\varphi_1\lambda + 2\left(\mathrm{ARL}_1 - 0.5\right)cnT_0\varphi_1\lambda + 2T_0W\varphi_1\lambda - bFT_0\lambda^2 \\ -cFnT_0\lambda^2 + bFT_0\varphi_1\lambda^2 + cFnT_0\varphi_1\lambda^2 + 2\mathrm{ARL}_0\left(b+cn\right)\left(1+F\lambda\right)\left(1+G\lambda\right) \end{bmatrix}.$$

The derivation of Equation (13) is shown in Yeong *et al.* (2012). In the process of obtaining the approximate optimal values n^*, k^*, h^*, we treat n and k as discrete variables and assume that the values of n are between 2 and 30, and the values of k are in $\{0.01, 0.02, \ldots, 3.00\}$. Note that only integer values of n are used. The upper limit of k is fixed as 3 because the minimum cost is always achieved before $k = 3$, while the upper limit of n is fixed as 30 as this is a reasonably large sample size, and a sample size larger that 30 is usually not preferred by practitioners. A numerical procedure using the Scicoslab software is employed to obtain the approximate optimal values n^*, k^* and h^* of the design parameters n, k and h, respectively as follows:

(1) Start with the combination (n, k) = (2, 0.01), i.e. by considering the lower bounds of each of the design parameters.
(2) Calculate ARL_1 and ARL_0 from Equation (11).
(3) Calculate h from Equation (13).
(4) Calculate the expected cost per hour, C from Equation (12).
(5) Increase k by 0.01, and maintain the values of n.
(6) Repeat Steps (2) and (5) until $k = 3$.
(7) Reset k to 0.01, and increase n by 1.
(8) Repeat Steps (2) to (7) until $n = 30$.

For the economic design, the parameters n^*, k^*, and h^* which give the overall approximate minimum cost per hour C^* are the approximate optimal design parameters for the CV chart. However, in the economic-statistical design, we minimize Equation (12), subject to two additional constraints: $\text{ARL}_0 \geq \text{LB}$ and $\text{ARL}_1 \leq \text{UB}$, where LB and UB are the desired lower and upper bounds on the ARL_0 and ARL_1, respectively. This is achieved by employing the same algorithm in Steps (1) to (8), but by imposing a large penalty cost to the cost function, if at least one of the ARL constraints is not satisfied. This is to filter out the design parameters that are not satisfying the ARL constraints. In this paper, the LB is set as 250 while the UB is set as 20.

Numerical Examples

In this section, we illustrate the application of the methodology described in the previous section on numerical examples. Table 1 shows the values of the input parameters for each example. For each example, the optimal cost, chart's parameters and ARLs are obtained based on the methodology described in the previous section. Tables 2 to 4 show the results of the optimal cost, the optimal chart's parameters and the ARLs for the economic and economic-statistical designs of the CV chart for γ_0 = 0.05, 0.10 and 0.20, respectively.

Table 1. Input Parameters for the Numerical Examples

No.		λ	τ	C_0 ($)	C_1 ($)	*Y ($)*	*W ($)*	*b($)*	*c($)*	*e*	T_0	T_1	T_2	φ_1	φ_2
1	$\lambda 1$	**0.01**	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
2	$\lambda 2'$	**0.02**	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
3	$\lambda 3$	**0.04**	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
4	$\tau 1$	0.02	**1.25**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
5	$\tau 2'$	0.02	**1.50**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
6	$\tau 3$	0.02	**1.75**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
7	$\tau 4$	0.02	**2.00**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
8	$\tau 5$	0.02	**2.50**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
9	$C_0 1$	0.02	1.50	**57.12**	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
10	$C_0 2'$	0.02	1.50	**114.24**	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
11	$C_0 3$	0.02	1.50	**228.48**	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
12	$C_1 1$	0.02	1.50	114.24	**474.6**	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
13	$C_1 2'$	0.02	1.50	114.24	**949.2**	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
14	$C_1 3$	0.02	1.50	114.24	**1898.4**	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
15	$Y 1$	0.02	1.50	114.24	949.2	**488.7**	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
16	$Y 2'$	0.02	1.50	114.24	949.2	**977.4**	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
17	$Y 3$	0.02	1.50	114.24	949.2	**1954.8**	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
18	$W 1$	0.02	1.50	114.24	949.2	977.4	**488.7**	0	4.22	0.083	0.083	0.083	0.75	1	0
19	$W 2'$	0.02	1.50	114.24	949.2	977.4	**977.4**	0	4.22	0.083	0.083	0.083	0.75	1	0
20	$W 3$	0.02	1.50	114.24	949.2	977.4	**1954.8**	0	4.22	0.083	0.083	0.083	0.75	1	0
21	$b 1'$	0.02	1.50	114.24	949.2	977.4	977.4	**0**	4.22	0.083	0.083	0.083	0.75	1	0
22	$b 2$	0.02	1.50	114.24	949.2	977.4	977.4	**5**	4.22	0.083	0.083	0.083	0.75	1	0
23	$b 3$	0.02	1.50	114.24	949.2	977.4	977.4	**10**	4.22	0.083	0.083	0.083	0.75	1	0
24	$c 1$	0.02	1.50	114.24	949.2	977.4	977.4	0	**2.11**	0.083	0.083	0.083	0.75	1	0
25	$c 2'$	0.02	1.50	114.24	949.2	977.4	977.4	0	**4.22**	0.083	0.083	0.083	0.75	1	0
26	$c 3$	0.02	1.50	114.24	949.2	977.4	977.4	0	**8.44**	0.083	0.083	0.083	0.75	1	0

27	$e1$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	**0.042**	0.083	0.083	0.75	1	0
28	$e2'$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	**0.083**	0.083	0.083	0.75	1	0
29	$e3$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	**0.166**	0.083	0.083	0.75	1	0
30	$T_0 1$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	**0.042**	0.083	0.75	1	0
31	$T_0 2'$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	**0.083**	0.083	0.75	1	0
32	$T_0 3$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	**0.166**	0.083	0.75	1	0
33	$T_1 1$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	**0.042**	0.75	1	0
34	$T_1 2'$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	**0.083**	0.75	1	0
35	$T_1 3$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	**0.166**	0.75	1	0
36	$T_2 1$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	**0.375**	1	0
37	$T_2 2'$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	**0.75**	1	0
38	$T_2 3$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	**1.5**	1	0
39	$\varphi_1 1'$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**1**	**0**
40	$\varphi_1 2$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**0**	**0**
41	$\varphi_2 1$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**0**	**1**
42	$\varphi_2 2$	0.02	1.50	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**1**	**1**

Remark: " ' " specifies the examples where all of the input parameters remain unchanged from the original example.

Table 2. Optimal Cost, Chart Parameters and ARLs for the Economic and Economic-statistical Designs of the CV Chart for $\gamma_0 = 0.05$

No.	Economic Design						Economic-statistical Design					
	*n**	*k**	*h**	*C ($)*	ARL_0	ARL_1	*n**	*k**	*h**	*C ($)*	ARL_0	ARL_1
1	10	2.33	2.07	189.29	53.04	2.38	9	2.91	1.29	195.54	252.06	3.87
2	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
3	6	2.39	0.70	280.62	64.00	3.62	6	2.96	0.46	293.33	251.32	5.76
4	4	2.16	0.62	275.62	38.10	8.76	14	2.89	0.74	328.98	250.51	9.92
5	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
6	6	2.57	1.12	203.28	99.54	2.38	7	2.94	1.05	205.85	254.96	2.55
7	6	2.71	1.28	191.39	138.99	1.82	6	2.96	1.14	192.13	251.32	2.00
8	5	3.00	1.25	179.39	251.98	1.54	5	3.00	1.25	179.39	251.98	1.54
9	7	2.38	1.04	173.44	61.88	3.21	8	2.92	0.81	182.68	252.24	4.34
10	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
11	8	2.36	1.32	330.92	58.22	2.86	8	2.92	0.90	339.13	252.24	4.34
12	11	2.29	2.67	188.83	47.49	2.16	11	2.90	1.84	194.61	252.84	3.20
13	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
14	5	2.42	0.54	278.49	68.54	4.24	6	2.96	0.42	291.98	251.32	5.76
15	4	2.18	0.74	216.48	39.80	4.11	6	2.96	0.60	232.48	251.32	5.76
16	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
17	11	2.55	1.53	234.83	95.56	2.53	9	2.91	0.98	238.75	252.06	3.87
18	7	2.38	1.07	217.06	61.88	3.21	8	2.92	0.83	226.01	252.24	4.34
19	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
20	7	2.38	1.09	244.03	61.88	3.21	8	2.92	0.84	252.81	252.24	4.34
21	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
22	10	2.28	1.63	229.70	46.51	2.31	11	2.92	1.21	239.99	252.54	3.25

23	11	2.23	1.89	232.49	40.60	2.09	14	2.98	1.60	243.64	250.51	2.56
24	8	2.6	0.78	209.32	109.45	3.39	8	2.92	0.61	211.85	252.24	4.34
25	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
26	5	2.18	1.26	246.88	38.74	3.53	8	2.92	1.18	268.14	252.24	4.34
27	13	2.28	1.93	220.57	45.88	1.91	14	2.89	1.47	229.32	250.51	2.56
28	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
29	5	2.41	0.82	232.39	67.00	4.21	5	3.00	0.53	241.92	251.98	6.94
30	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
31	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
32	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
33	7	2.38	1.08	225.48	61.88	3.21	8	2.92	0.83	234.38	252.24	4.34
34	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
35	7	2.38	1.08	227.20	61.88	3.21	8	2.92	0.84	236.09	252.24	4.34
36	7	2.38	1.08	227.62	61.88	3.21	8	2.92	0.83	236.57	252.24	4.34
37	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
38	7	2.38	1.08	222.97	61.88	3.21	8	2.92	0.83	231.77	252.24	4.34
39	7	2.38	1.08	226.05	61.88	3.21	8	2.92	0.83	234.95	252.24	4.34
40	7	2.38	1.08	224.30	61.88	3.21	8	2.92	0.83	233.36	252.24	4.34
41	7	2.37	1.10	237.76	60.30	3.18	8	2.92	0.85	246.92	252.24	4.34
42	7	2.38	1.09	239.52	61.88	3.21	8	2.92	0.85	248.51	252.24	4.34

Table 3. Optimal Cost, Chart Parameters and ARLs for the Economic and Economic-statistical Designs of the CV Chart for $\gamma_0 = 0.10$

No.	Economic Design						Economic-statistical Design					
	*n**	*k**	*h**	*C ($)*	ARL_0	ARL_1	*n**	*k**	*h**	*C ($)*	ARL_0	ARL_1
1	10	2.33	2.06	189.58	52.70	2.40	11	2.91	1.57	196.17	250.82	3.25
2	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
3	6	2.39	0.70	281.16	62.95	3.64	7	2.96	0.54	294.70	254.43	5.05
4	4	2.16	0.62	275.77	37.65	8.73	15	2.90	0.79	331.18	252.42	9.46
5	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
6	6	2.57	1.12	203.79	96.73	2.40	7	2.96	1.04	206.55	254.43	2.59
7	6	2.71	1.28	191.90	133.75	1.83	6	2.99	1.13	192.76	255.24	2.04
8	5	3.01	1.25	179.85	242.98	1.56	5	3.03	1.24	179.86	254.36	1.57
9	7	2.38	1.04	173.89	61.08	3.22	8	2.94	0.80	183.63	253.51	4.42
10	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
11	8	2.36	1.32	331.35	57.64	2.88	8	2.94	0.89	339.98	253.51	4.42
12	11	2.28	2.68	189.16	46.05	2.17	11	2.91	1.83	195.09	250.82	3.25
13	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
14	6	2.41	0.63	279.13	66.08	3.69	6	2.99	0.41	293.64	255.24	5.89
15	4	2.17	0.75	216.73	38.46	4.09	7	2.96	0.70	233.50	254.43	5.05
16	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
17	11	2.55	1.52	235.48	94.23	2.55	11	2.91	1.19	239.60	250.82	3.25
18	7	2.38	1.07	217.49	61.08	3.22	8	2.94	0.82	226.93	253.51	4.42
19	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
20	7	2.38	1.09	244.46	61.08	3.22	8	2.94	0.84	253.71	253.51	4.42
21	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
22	10	2.28	1.62	230.21	46.27	2.33	11	2.91	1.20	240.80	250.82	3.25

23	11	2.23	1.88	233.04	40.47	2.11	14	2.90	1.59	244.56	250.91	2.60
24	8	2.60	0.78	209.77	106.96	3.41	8	2.94	0.60	212.52	253.51	4.42
25	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
26	5	2.18	1.26	247.22	38.44	3.54	8	2.94	1.17	269.38	253.51	4.42
27	13	2.28	1.92	221.12	45.71	1.93	14	2.90	1.45	230.20	250.91	2.60
28	7	2.38	1.09	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
29	5	2.41	0.83	232.81	65.51	4.21	5	3.03	0.52	242.78	254.36	7.07
30	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
31	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
32	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
33	7	2.38	1.08	225.92	61.08	3.22	8	2.94	0.82	235.29	253.51	4.42
34	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
35	7	2.38	1.08	227.63	61.08	3.22	8	2.94	0.83	237.00	253.51	4.42
36	7	2.38	1.08	228.06	61.08	3.22	8	2.94	0.83	237.49	253.51	4.42
37	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
38	7	2.38	1.08	223.40	61.08	3.22	8	2.94	0.82	232.67	253.51	4.42
39	7	2.38	1.08	226.49	61.08	3.22	8	2.94	0.83	235.86	253.51	4.42
40	7	2.38	1.08	224.74	61.08	3.22	8	2.94	0.83	234.27	253.51	4.42
41	7	2.37	1.10	238.19	59.56	3.20	8	2.94	0.84	247.83	253.51	4.42
42	7	2.38	1.10	239.95	61.08	3.22	8	2.94	0.84	249.42	253.51	4.42

Table 4. Optimal Cost, Chart Parameters and ARLs for the Economic and Economic-statistical Designs of the CV Chart for $\gamma_0 = 0.20$

No.	Economic Design						Economic-statistical Design					
	*n**	*k**	*h**	*C ($)*	ARL_0	ARL_1	*n**	*k**	*h**	*C ($)*	ARL_0	ARL_1
1	9	2.34	1.85	191.04	52.65	2.69	10	2.98	1.37	198.79	252.45	3.82
2	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
3	6	2.38	0.71	283.44	57.67	3.67	6	3.08	0.45	298.93	250.78	6.26
4	4	2.13	0.66	276.52	33.94	8.33	15	2.94	0.77	335.50	252.82	9.93
5	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
6	7	2.55	1.29	205.84	85.43	2.18	6	3.08	0.86	209.45	250.78	3.22
7	6	2.72	1.26	193.96	118.13	1.91	6	3.08	1.08	195.13	250.78	2.18
8	4	3.17	0.97	181.69	231.67	1.97	4	3.21	0.96	181.70	250.63	1.99
9	7	2.37	1.05	175.76	56.64	3.27	6	3.08	0.58	187.40	250.78	6.62
10	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
11	7	2.37	1.18	333.00	56.64	3.27	8	3.02	0.86	343.37	254.42	4.75
12	10	2.29	2.42	190.41	46.44	2.42	10	2.98	1.60	197.38	252.45	3.82
13	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
14	6	2.4	0.63	281.82	60.25	3.72	6	3.08	0.40	298.33	250.78	6.26
15	4	2.15	0.77	217.81	35.25	4.06	4	3.21	0.39	236.39	250.63	9.25
16	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
17	10	2.56	1.39	238.07	90.65	2.86	10	2.98	1.04	243.26	252.45	3.82
18	7	2.37	1.08	219.30	56.64	3.27	8	3.02	0.79	230.62	254.42	4.75
19	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
20	7	2.37	1.10	246.23	56.64	3.27	8	3.02	0.80	257.34	254.42	4.75
21	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
22	9	2.28	1.47	232.19	45.46	2.59	11	2.97	1.16	244.65	253.71	3.94

23	11	2.22	1.86	235.17	38.94	2.18	13	2.95	1.42	248.35	251.85	2.98
24	8	2.61	0.78	211.65	99.89	3.51	8	3.02	0.58	215.28	254.42	4.75
25	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
26	5	2.16	1.28	248.68	35.65	3.55	6	3.08	0.84	274.28	250.78	6.26
27	12	2.28	1.76	223.26	45.09	2.12	13	2.95	1.29	233.79	251.85	2.98
28	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
29	5	2.4	0.85	234.59	58.87	4.21	4	3.21	0.41	245.34	250.64	9.25
30	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
31	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
32	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
33	7	2.37	1.09	227.72	56.64	3.27	8	3.02	0.79	238.97	254.42	4.75
34	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
35	7	2.37	1.09	229.42	56.64	3.27	8	3.02	0.80	240.67	254.42	4.75
36	7	2.37	1.09	229.87	56.64	3.27	8	3.02	0.80	241.18	254.42	4.75
37	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
38	7	2.37	1.09	225.18	56.64	3.27	8	3.02	0.79	236.30	254.42	4.75
39	7	2.37	1.09	228.28	56.64	3.27	8	3.02	0.79	239.53	254.42	4.75
40	7	2.37	1.08	226.51	56.64	3.27	8	3.02	0.79	237.94	254.42	4.75
41	7	2.36	1.10	239.95	55.34	3.25	8	3.02	0.81	251.48	254.42	4.75
42	7	2.37	1.10	241.73	56.64	3.27	8	3.02	0.81	253.07	254.42	4.75

There are 14 input parameters $\left(\lambda, \tau, C_0, C_1, Y, W, b, c, e, T_0, T_1, T_2, \varphi_1 \text{ and } \varphi_2\right)$ related to the cost function of the economic and economic-statistical designs of the CV chart. In this section, we want to identify the impact of these input parameters towards the optimal cost (C), optimal chart's parameters (h^*, n^* and k^*) and ARLs (ARL_0 and ARL_1) based on Tables 2 to 4.

λ , which is the process failure rate, has significant effect on the optimal sampling interval (h^*), cost (C), and optimal sample size (n^*). When the failure rate increase, a smaller sampling interval is required, which means more frequent samplings are needed to minimize cost due to the increased need to monitor the process with a higher failure rate. A smaller sample size is needed when the failure rate increases as a smaller sample size is more economically desirable to compensate for the higher sampling cost. The expected hourly cost (C) also increases due to the expected increase in out-of-control conditions and an increase in the sampling cost.

An increase in the shift in the CV (τ) results in a decrease in the average run length while out of control (ARL_1), as larger shifts are easier to detect. For larger τ , a larger optimal sampling interval (h^*) is more cost effective due to the ease in detecting larger shifts, thus less frequent sampling will be done. Besides that, as τ increases, a smaller sample size is needed and this results in a decrease in the cost.

The quality cost per hour while in control (C_0) and out-of-control (C_1) have significant effect on the optimal sampling interval (h^*) and the cost (C). The increase in C_0 results in an increase in h^* and C^*. In this case, the sampling should be done less frequently and there is a slight increase in the sample size. This is because when C_0 increases, it is not economically desirable to perform frequent sampling for a fast detection of the assignable cause. On the other hand, an increase in C_1 results in a decrease in h^* but an increase in C^*. Due to the increased frequency of sampling, a smaller sample size should be adopted. More frequent sampling is required for larger C_1, as faster detection is preferable to prevent the process to be in an out-of-control process for too long. C_0 has more obvious effect on the cost compared to C_1.

The increase in the cost of false alarm (Y) and the cost of finding and fixing an assignable cause (W) results in a slight increase in the cost (C). W has no significant effect on the chart's parameters. However, W has larger effect on the cost compare to Y. When Y increases, it results in higher optimal sample size (n^*) and higher optimal sampling interval (h^*). The less frequent sampling is needed to compensate for the increase in the cost due to a larger n. Increase in Y also results in a higher ARL_0. This shows that there are fewer false alarms.

An increase in the fixed cost per sample (b) and cost per unit sampled (c) result in an increase in cost (C) and decrease in optimal control limit coefficient (k^*). However, c has larger effect on the cost compared to b. The increase in both b and c will result in a larger optimal sampling interval (h^*), which means less frequent sampling should be done. However, larger sample size should be taken for a larger b but smaller sample size should be taken for a larger c. Increase in b and c result in a lower ARL_0.

The expected time to sample and interpret one unit (e) has significant effect on cost, optimal sample size (n^*) and optimal sampling interval (h^*). Increase in e results in a smaller sampling interval. More frequent sampling results in an increase in cost. So, smaller sample size is required when e increases to compensate for the high cost due to more frequent sampling. Besides that, the increase in e also results in a larger average run length when out-of-control (ARL_1).

The expected time to find and repair the assignable cost (T_1 and T_2) has only slight effect towards the optimal chart parameters, cost and ARLs. Lastly, whether the production continues or stops during search or repair have only slight effects on the cost and other chart's parameters.

Next, we look at the impact of γ_0 towards the optimal cost and chart's parameters. γ_0 has minimal effect towards k^*, h^*, cost and ARLs. Increase in γ_0 results in a larger optimal k^*. A smaller sampling interval was needed when γ_0 increase and this leads to a higher cost. Besides that, increase in γ_0 results in a slight increase in ARL_0 and ARL_1.

Subsequently, the effects of adding statistical constraints are investigated. There is a slight increase in cost after adding the statistical constraints, but a significantly larger ARL_0 was obtained. Table 5 shows the percentage increase in the optimal cost when statistical constraints are added; while Table 6 shows the percentage increase in the ARL_0. From Tables 5 and 6, an average 3.4% increase in cost and an average 74% increase in ARL_0 are shown. This shows that the small increase in cost is compensated by a large reduction in the false alarm rate. Although a slightly larger ARL_1 was obtained, the large improvement in ARL_0 more than compensates for the increase in ARL_1. There are some changes in choosing the chart's parameters when statistical constraints are added. A smaller h^* and a larger k^* were obtained in economic-statistical designs. Furthermore, a larger optimal sample size n^* is needed when statistical constraints are added.

Table 5. Percentage Increase in the Optimal Cost when Statistical Constraints are Added

No.	$\gamma_0 = 0.05$			$\gamma_0 = 0.10$			$\gamma_0 = 0.20$		
	Economic Design	Economic Statistical Design	% increase	Economic Design	Economic Statistical Design	% increase	Economic Design	Economic Statistical Design	% increase
1	189.29	195.54	3.20	189.58	196.17	3.36	191.04	198.79	3.90
2	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
3	280.62	293.33	4.33	281.16	294.7	4.59	283.44	298.93	5.18
4	275.62	328.98	16.22	275.77	331.18	16.73	276.52	335.5	17.58
5	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
6	203.28	205.85	1.25	203.79	206.55	1.34	205.84	209.45	1.72
7	191.39	192.13	0.39	191.9	192.76	0.45	193.96	195.13	0.60
8	179.39	179.39	0.00	179.85	179.86	0.01	181.69	181.7	0.01
9	173.44	182.68	5.06	173.89	183.63	5.30	175.76	187.4	6.21
10	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
11	330.92	339.13	2.42	331.35	339.98	2.54	333	343.37	3.02
12	188.83	194.61	2.97	189.16	195.09	3.04	190.41	197.38	3.53
13	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70

14	278.49	291.98	4.62	279.13	293.64	4.94	281.82	298.33	5.53
15	216.48	232.48	6.88	216.73	233.5	7.18	217.81	236.39	7.86
16	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
17	234.83	238.75	1.64	235.48	239.6	1.72	238.07	243.26	2.13
18	217.06	226.01	3.96	217.49	226.93	4.16	219.3	230.62	4.91
19	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
20	244.03	252.81	3.47	244.46	253.71	3.65	246.23	257.34	4.32
21	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
22	229.7	239.99	4.29	230.21	240.8	4.40	232.19	244.65	5.09
23	232.49	243.64	4.58	233.04	244.56	4.71	235.17	248.35	5.31
24	209.32	211.85	1.19	209.77	212.52	1.29	211.65	215.28	1.69
25	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
26	246.88	268.14	7.93	247.22	269.38	8.23	248.68	274.28	9.33
27	220.57	229.32	3.82	221.12	230.2	3.94	223.26	233.79	4.50
28	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
29	232.39	241.92	3.94	232.81	242.78	4.11	234.59	245.34	4.38
30	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
31	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
32	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
33	225.48	234.38	3.80	225.92	235.29	3.98	227.72	238.97	4.71
34	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
35	227.2	236.09	3.77	227.63	237	3.95	229.42	240.67	4.67
36	227.62	236.57	3.78	228.06	237.49	3.97	229.87	241.18	4.69
37	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
38	222.97	231.77	3.80	223.4	232.67	3.98	225.18	236.3	4.71
39	226.05	234.95	3.79	226.49	235.86	3.97	228.28	239.53	4.70
40	224.3	233.36	3.88	224.74	234.27	4.07	226.51	237.94	4.80
41	237.76	246.92	3.71	238.19	247.83	3.89	239.95	251.48	4.58
42	239.52	248.51	3.62	239.95	249.42	3.80	241.73	253.07	4.48

Table 6. Percentage Increase in ARL_0 when Statistical Constraints are Added

No.	$\gamma_0 = 0.05$			$\gamma_0 = 0.10$			$\gamma_0 = 0.20$		
	Economic Design	Economic Statistical Design	% increase	Economic Design	Economic Statistical Design	% increase	Economic Design	Economic Statistical Design	% increase
1	53.04	252.06	78.96	52.7	250.82	78.99	52.65	252.45	79.14
2	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
3	64	251.32	74.53	62.95	254.43	75.26	57.67	250.78	77.00
4	38.1	250.51	84.79	37.65	252.42	85.08	33.94	252.82	86.58
5	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
6	99.54	254.96	60.96	96.73	254.43	61.98	85.43	250.78	65.93
7	138.99	251.32	44.70	133.75	255.24	47.60	118.13	250.78	52.89
8	251.98	251.98	0.00	242.98	254.36	4.47	231.67	250.63	7.56
9	61.88	252.24	75.47	61.08	253.51	75.91	56.64	250.78	77.41
10	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
11	58.22	252.24	76.92	57.64	253.51	77.26	56.64	254.42	77.74

12	47.49	252.84	81.22	46.05	250.82	81.64	46.44	252.45	81.60
13	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
14	68.54	251.32	72.73	66.08	255.24	74.11	60.25	250.78	75.97
15	39.8	251.32	84.16	38.46	254.43	84.88	35.25	250.63	85.94
16	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
17	95.56	252.06	62.09	94.23	250.82	62.43	90.65	252.45	64.09
18	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
19	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
20	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
21	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
22	46.51	252.54	81.58	46.27	250.82	81.55	45.46	253.71	82.08
23	40.6	250.51	83.79	40.47	250.91	83.87	38.94	251.85	84.54
24	109.45	252.24	56.61	106.96	253.51	57.81	99.89	254.42	60.74
25	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
26	38.74	252.24	84.64	38.44	253.51	84.84	35.65	250.78	85.78
27	45.88	250.51	81.69	45.71	250.91	81.78	45.09	251.85	82.10
28	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
29	67	251.98	73.41	65.51	254.36	74.25	58.87	250.64	76.51
30	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
31	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
32	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
33	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
34	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
35	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
36	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
37	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
38	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
39	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
40	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74
41	60.3	252.24	76.09	59.56	253.51	76.51	55.34	254.42	78.25
42	61.88	252.24	75.47	61.08	253.51	75.91	56.64	254.42	77.74

Conclusions

The economic and economic-statistical designs of the CV chart are studied in this paper. Prior to this paper, the economic and economic-statistical designs of the CV chart cannot be found in the existing literature. The CV chart is used to monitor processes where the mean and/or variance is not constant, but the ratio of the standard deviation against the mean is constant. Through the economic design, practitioners will be able to choose the chart's parameters of the CV chart in order to minimize the cost; while the economic-statistical design allows practitioners to minimize the cost subject to constraints in the in-control and out-of-control average run lengths. Sensitivity analysis is performed on both the economic and economic-statistical designs of the CV chart. The sensitivity analysis is used to determine which input parameters have a significant effect towards the optimal cost, choices of chart's parameter and average run length (ARLs). The input parameters that have significant effect towards the optimal cost are the shift size (τ), the process failure rate (λ), the quality cost per hour while in control (C_0) and out-of-control (C_1), and the expected time to sample and interpret one unit (e); while the expected search time for a false alarm (T_0) has no effect towards the optimal cost. Besides the impact of the input parameters, the impact of the in-control CV (γ_0) towards the optimal cost, choice of chart's parameters and *ARL*s are also investigated. Lastly, the impact of adding statistical constraints in the economic design is studied. It was found that adding statistical constraints result in a slight increase in cost and ARL_1 but result in a significantly larger ARL_0. This shows that there is a large reduction in the false alarm rate, with only a slight increase in cost.

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