Academic Journal of Science,
CD-ROM. ISSN: 2165-6282 :: 05(01):181–188 (2016)

SYNCHRONIZATION OF CHAOTIC SUPPLY CHAIN SYSTEMS USING A SINGLE CONTROLLER BASED ON THE LYAPUNOV FUNCTION

Alper Göksu, Uğur Erkin Kocamaz, Yılmaz Uyaroğlu and Harun Taşkın

Sakarya University, Turkey

In this paper, a single state controller is designed for the synchronization of two identical chaotic supply chain systems. A Lyapunov function is used for achieving the global asymptotical stability of synchronization errors by ensuring a negative value of its time derivative. Computer simulation results are demonstrated to validate the theoretical analysis. Furthermore, they show that the proposed controller is very effective for the synchronization of chaotic supply chain systems.

Keywords: Chaos synchronization, Chaotic supply chain system, Single controller, Lyapunov function.

Introduction

Some of the nonlinear systems include chaotic behaviours in their dynamics. In 1963, when Edward Lorenz was using a model for fluid conviction that describes some features of the atmospheric processes, he noticed that this differential system has very sensitive dependence on initial conditions [1]. It was the first time chaos has been observed mathematically. Since then, chaos has been intensively studied and the Lorenz attractor has become the famous chaotic system. In addition, new chaotic systems have been discovered in a variety of fields including physics [2], chemistry [3], ecology [4], biology [5] and finance [6].

The chaotic systems also have an infinite number of different periodic trajectories. Sometimes, it is desired that two dynamical systems have the same values. Even if they are identical, the chaotic systems can lead to completely different trajectories due to the slight errors. So, the synchronization of them is required. At first, it was believed that the chaotic systems cannot be synchronized because of the sensitivity on the initial conditions. In 1990, Pecora and Carroll successfully applied the synchronization of two identical chaotic systems which are starting from different initial conditions [7]. Afterwards, chaos synchronization has also become an important task. Several effective control methods are proposed for the chaos synchronization. For instance, active control [8], sliding mode control [9], passive control [10], backstepping design [11] and impulsive control [12] are used for the synchronization of chaotic systems. In these methods, the stability of synchronization error system is generally established with a Lyapunov function. The direct Lyapunov stability theory can also be used for the synchronization of chaotic systems.

The supply chain can be defined as "a way to envision all steps needed from beginning to end in order to deliver products or services to the customer" [13]. The supply chain involves all stages from beginning to end; which in the manufacturing industry is often presented as "from raw materials to distributed products or services" [13, 14]. The receipt of raw materials from the manufacturing processes to the distribution and the delivery of products to customers includes enterprise organizational functions [15]. The supply chain system also enables the manufacturing enterprise organizations to achieve higher quality products, lower inventory quantities, and better customer services. Researchers have paid attentions to the special issues of supply chain system analyzing, designing and modelling in recent years.

There are some unpredictable factors in the supply chain dynamics and they impact the effectiveness of the system. Supply chain planning, managing and scheduling may lead to some nonlinear behaviours in system components, production and inventory levels under different stages. The demand forecast updating, price fluctuation, order batching, rationing and shortages are the major issues [14]. In addition, the usage of demand processing and non-zero lead times are the other important topics in the supply chain management [16]. However, they have resulted in nonlinearities and chaotic activities [16–22]. For example, the bullwhip effect, which is one of the undesirable behaviours in the supply chain system, causes it as a chaotic system [23].

Chaotic cases at inventory levels and in production control mechanisms are the unwanted problems. The synchronization of supply chain systems leads the correct demand information might be retrieved by enterprise resource planning system on time even if it has chaotic trajectories. There are a few studies about the synchronization of chaotic supply chains in the literature. In 2006, the radial basis function neural networks were used for the chaos synchronization of bullwhip effect in a supply chain system [23]. The synchronization of chaos in supply chains was achieved with the adaptive feedback control model to mitigate the effects due to uncertainties and perturbations in 2009 [24]. Recently, the synchronization of chaotic supply chain systems has been applied by means of the active control [25] and adaptive neuro-fuzzy inference system methods [26]. In this study, the synchronization of two identical chaotic supply chain systems with a single state controller is investigated. The Lyapunov stability analysis is used for providing the stability of synchronization errors. Numerical simulations are presented to verify the effectiveness of synchronization results.

Chaotic Supply Chain System

A supply chain system contains three levels: end customers, distributors, and producers. The dynamic equations of a supply chain system can be described by [24]:

$$\begin{aligned} \dot{x} &= (m+\delta m)y-(n+1+\delta n)x+d_1, \\ \dot{y} &= (r+\delta r)x-y-xz+d_2, \\ \dot{z} &= xy+(k-1-\delta k)z+d_3, \end{aligned} \tag{1}$$

where x, y and z are demand, inventory and produced quantities; m, n, r and k are the delivery efficiency of distributors, the ratio of customer demand, distortion coefficient and safety stock coefficient; δm, δn, δr and δk are the linear distortions of system parameters m, n, r and k; d_1, d_2 and d_3 are nonlinear perturbations in the different levels of system from outside, respectively [24].

With specific values set at $m = 10$, $n = 9$, $r = 28$, $k = -5/3$, $\delta m = 0.1$, $\delta n = 0.1$, $\delta r = 0.2$, $\delta k = 0.3$, $d_1 = 0.2\sin(t)$, $d_2 = 0.1\cos(5t)$ and $d_3 = 0.3\sin(t)$, the dynamic behaviour of supply chain system (1) exhibits an irregular motion with the initial conditions of $(x, y, z) = (0, 0.11, 9)$. In Fig. 1, Fig. 2 and Fig. 3, the trajectories of supply chain system (1) are shown as the states of chaotic motion in time series, two-dimensional phase portraits and three-dimensional phase plane, respectively.

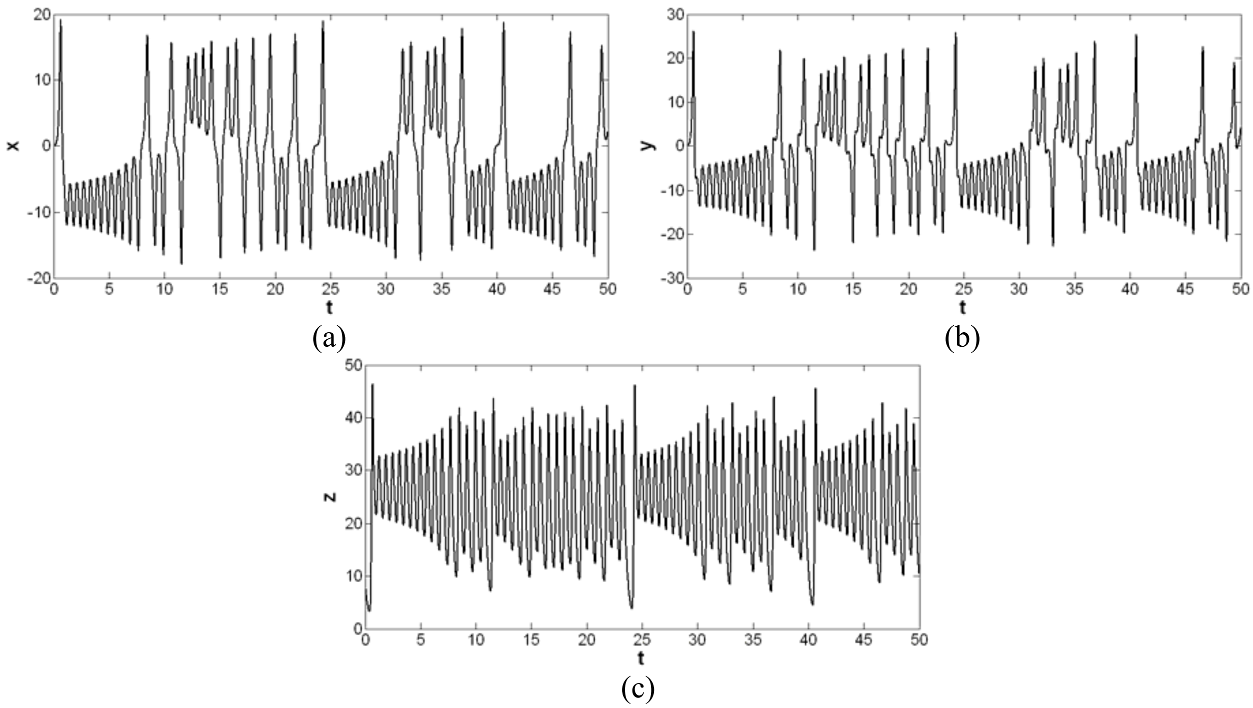


Figure 1. Chaotic supply chain system in time series (a) *x*, (b) *y*, (c) *z* signals

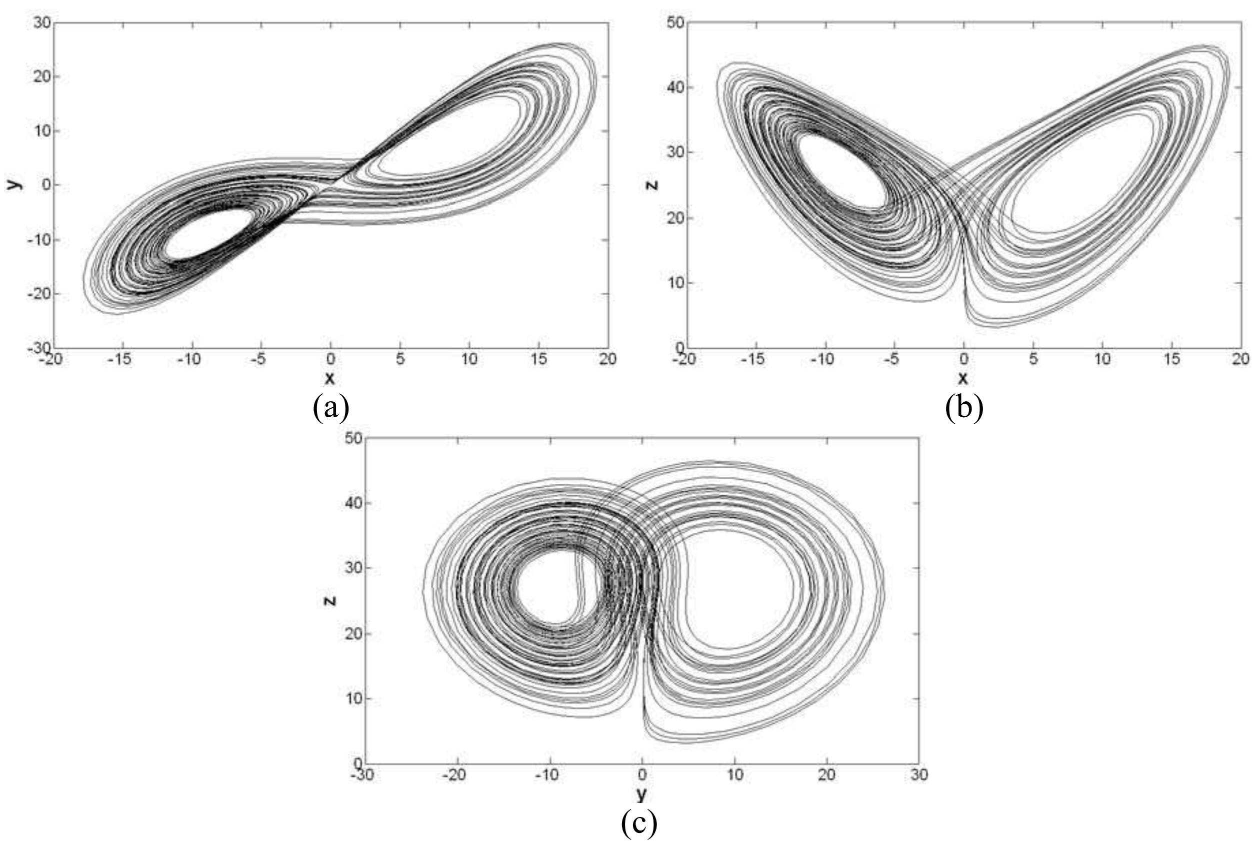


Figure 2. Chaotic supply chain system in 2D phase portraits (a) *x*–*y*, (b) *x*–*z*, (c) *y*–*z* phase plot

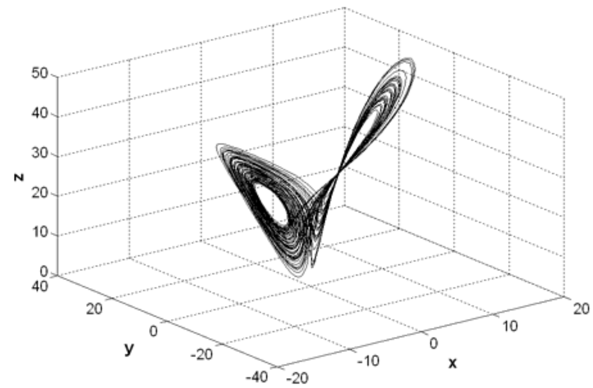


Figure 3. Chaotic supply chain system in 3D phase plane

Synchronization

For the synchronization, two coupled chaotic supply chain systems are considered with different initial conditions. The drive and response systems are denoted by subscript 1 and subscript 2, respectively. They are expressed as follows:

$$\begin{aligned} \dot{x}_1 &= (m+\delta m)y_1 - (n+1+\delta n)x_1 + 0.2\sin(t), \\ \dot{y}_1 &= (r+\delta r)x_1 - y_1 - x_1 z_1 + 0.1\cos(5t), \\ \dot{z}_1 &= x_1 y_1 + (k-1-\delta k)z_1 + 0.3\sin(t), \end{aligned} \tag{2}$$

and

$$\begin{aligned} \dot{x}_2 &= (m+\delta m)y_2 - (n+1+\delta n)x_2 + 0.2\sin(t) + u, \\ \dot{y}_2 &= (r+\delta r)x_2 - y_2 - x_2 z_2 + 0.1\cos(5t), \\ \dot{z}_2 &= x_2 y_2 + (k-1-\delta k)z_2 + 0.3\sin(t) \end{aligned} \tag{3}$$

where u in system (3) is the control function to be determined. To obtain the control function for the synchronization, the drive system is subtracted from the response system. So, the state errors e_1, e_2, and e_3 are defined as

$$\begin{aligned} e_1 &= x_2 - x_1, \\ e_2 &= y_2 - y_1, \\ e_3 &= z_2 - z_1. \end{aligned} \tag{4}$$

This leads to

$$\begin{aligned} \dot{e}_1 &= (m+\delta m)e_2 - (n+1+\delta n)e_1 + u, \\ \dot{e}_2 &= (r+\delta r)e_1 - e_2 - x_2 z_2 + x_1 z_1, \\ \dot{e}_3 &= x_2 y_2 - x_1 y_1 + (k-1-\delta k)e_3. \end{aligned} \tag{5}$$

The system (5) is called the error system. Some terms of system (5) can be written as

$$\begin{aligned} -x_2 z_2 + x_1 z_1 &= -z_2 e_1 - x_1 e_3. \\ x_2 y_2 - x_1 y_1 &= y_2 e_1 + x_1 e_2. \end{aligned} \tag{6}$$

This implies that the error system (5) can be rewritten as

$$\begin{aligned} \dot{e}_1 &= (m+\delta m)e_2 - (n+1+\delta n)e_1 + u, \\ \dot{e}_2 &= (r+\delta r)e_1 - e_2 - z_2 e_1 - x_1 e_3, \\ \dot{e}_3 &= y_2 e_1 + x_1 e_2 + (k-1-\delta k)e_3. \end{aligned} \tag{7}$$

Now, the goal is to ensure the error system asymptotically stable at the origin for the synchronization. Let the Lyapunov function for system (7) is selected as follows:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2). \tag{8}$$

It is clear that Lyapunov function (8) has positive definite and it is equal to zero at the equilibrium of system (7). Furthermore, the time derivative of the Lyapunov function V is

$$\begin{aligned} \dot{V} &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 \\ &= [(m+\delta m)e_2 - (n+1+\delta n)e_1 + u]e_1 + [(r+\delta r)e_1 - e_2 - z_2 e_1 - x_1 e_3]e_2 \\ &\quad + [y_2 e_1 + x_1 e_2 + (k-1-\delta k)e_3]e_3 \\ &= -(n+1+\delta n)e_1^2 - e_2^2 + (k-1-\delta k)e_3^2 \\ &\quad + [(m+\delta m + r + \delta r - z_2)e_2 + y_2 e_3]e_1 + ue_1. \end{aligned} \tag{9}$$

There are many alternatives for the u controller. If it is simply considered as

$$u = -(m+\delta m + r + \delta r - z_2)e_2 - y_2 e_3, \tag{10}$$

then $\dot{V}$ becomes

$$\dot{V} = -(n+1+\delta n)e_1^2 - e_2^2 + (k-1-\delta k)e_3^2, \tag{11}$$

where it has negative definite due to the parameter values $n = 9$, $\delta n = 0.1$, $k = -5/3$, and $\delta k = 0.3$. As a consequence, from Lyapunov direct method, the zero solution of the system (7) is asymptotically stable with the choice of Eq. (10). This implies that the synchronization of two identical supply chain systems is achieved.

Numerical Simulations

In this section, the synchronization of two identical chaotic supply chain systems is performed with the computer simulations and the results are demonstrated. In the simulations, fourth-order Runge–Kutta method is used with the variable step size. To ensure the chaotic behaviour of supply chain systems, the parameters are taken as $m = 10$, $n = 9$, $r = 28$, $k = -5/3$, $\delta m = 0.1$, $\delta n = 0.1$, $\delta r = 0.2$ and $\delta k = 0.3$. The initial values of supply chain systems are considered as $(x_1, y_1, z_1) = (0, 0.11, 9)$ and $(x_2, y_2, z_2) = (24, 20, 28)$. The controller is activated at $t = 10$ and $t = 20$, and the synchronization results are shown in Fig. 4 and Fig. 5, respectively. In Fig. 6, the error signals of synchronizations are shown.

As expected, the synchronization of chaotic supply chain systems are achieved in Fig. 4 and Fig. 5 after the controller is turned on, and the error signals which are demonstrated in Fig. 6 converge asymptotically to zero. Hence, these simulations confirm the validity of the proposed control technique. When the controller is activated at $t = 10$ and $t = 20$, the synchronization is completely observed at $t \geq 12$ and $t \geq 22$, respectively. So, the proposed Lyapunov function based single state controller is very effective for the synchronization of two identical chaotic supply chain systems.

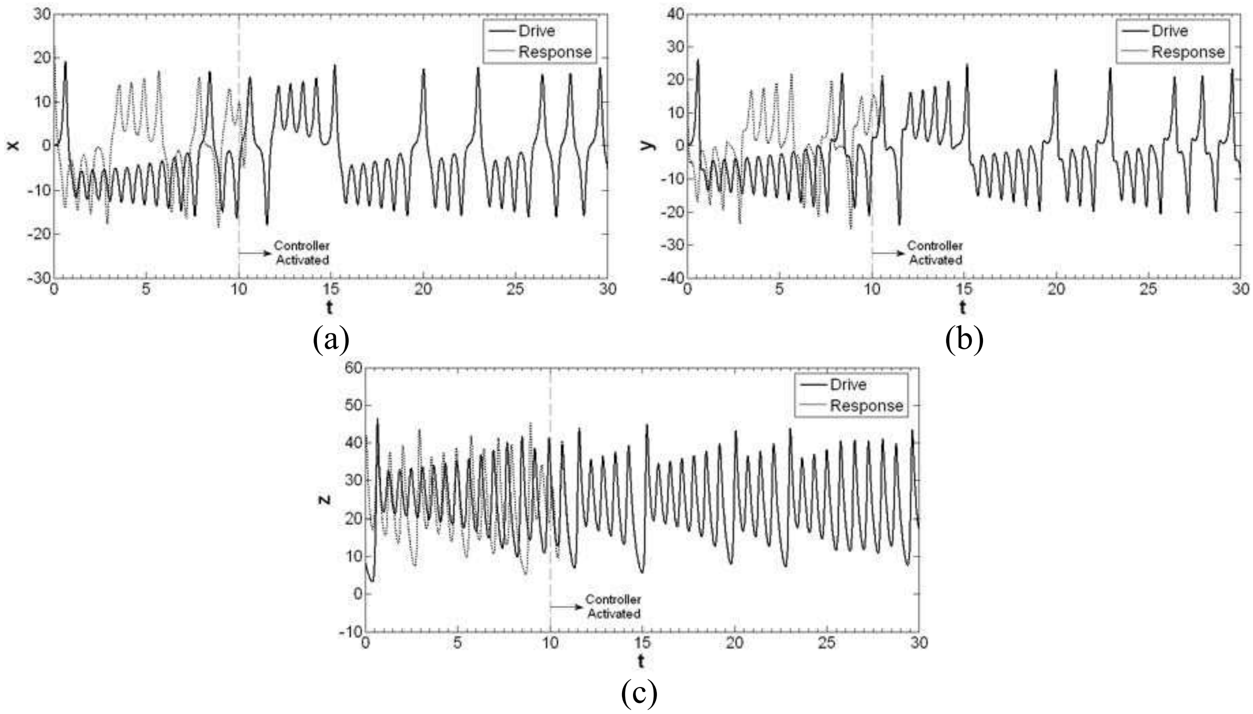


Figure 4. Chaotic supply chain systems in time series with the controller is activated at $t = 10$ (a) x, (b) y, (c) z signals

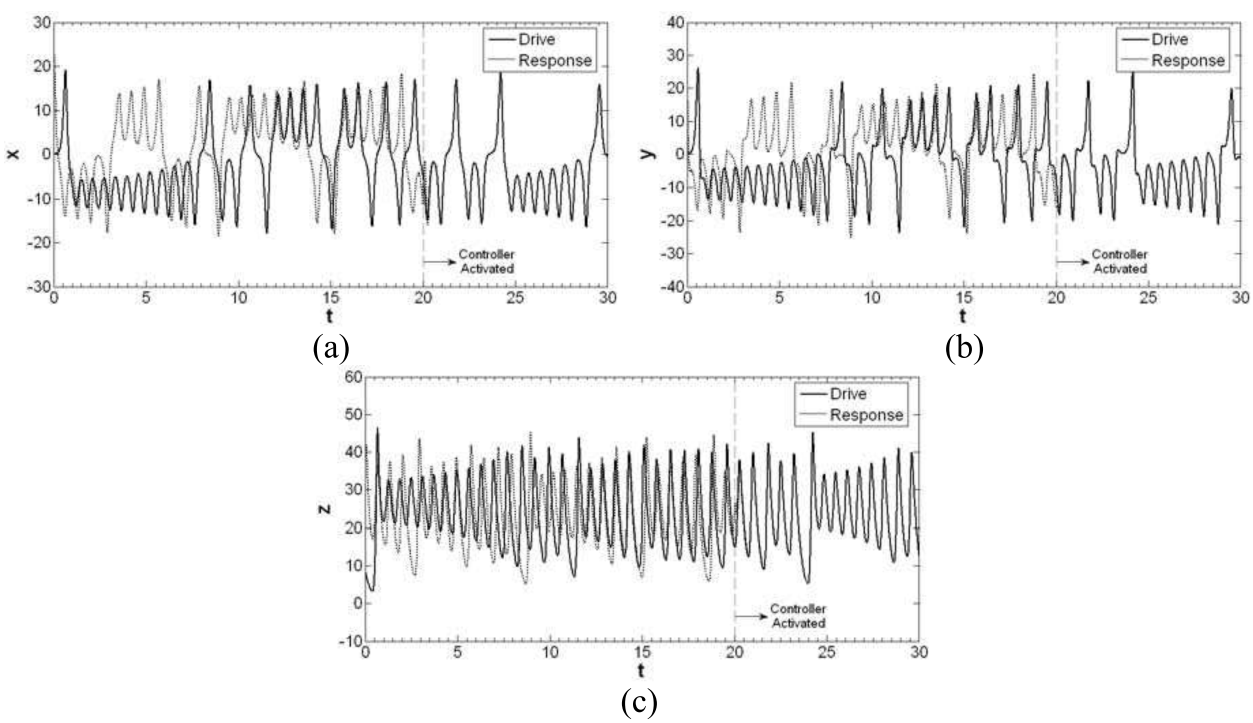
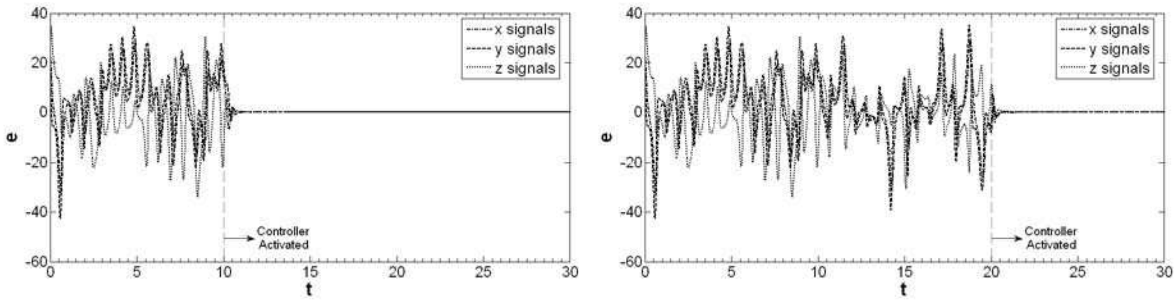


Figure 5. Chaotic supply chain systems in time series with the controller is activated at $t = 20$ (a) x, (b) y, (c) z signals



(a) (b)

Figure 6. The error signals of synchronization in time series with the controller is activated at (a) $t = 10$, (b) $t = 20$

Conclusion

In this paper, the synchronization of two identical chaotic supply chain systems is applied with a single state controller. Nowadays, it is aimed that supply chain management systems must be effective. But it is difficult to operate especially when they have distortions, nonlinear and chaotic behaviours. By synchronizing, the correct demand information can be retrieved by enterprise resource planning system on time and the effectiveness of system can be improved with appropriate synchronization and control mechanisms. Based on the Lyapunov stability theory, a controller is constructed and added to the response supply chain system for synchronization. Numerical simulations validate the theoretical analysis. Simulations also show that even through only a single state controller is used, the synchronization of chaotic supply chain systems is achieved in an appropriate time period.

References

1. Lorenz, E. N. (1963). Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, **20**(2), 130–141.
2. Heo, Y. S., Jung, J. W., Kim, J. M., Jo, M. K., & Song, H. J. (2012). Chaotic dynamics of a Chua's system with voltage controllability. *Journal of the Korean Physical Society*, **60**(7), 1140–1144.
3. Bodale, I., & Oancea, V. A. (2015). Chaos control for Willamowski-Rossler model of chemical reactions. *Chaos, Solitons & Fractals*, **78**, 1–9.
4. Das, K, Reddy, K. S., Srinivas, M. N., & Gazi, N. H. (2014). Chaotic dynamics of a three species prey-predator competition model with noise in ecology. *Applied Mathematics and Computation*, **231**, 117–133.
5. El-Gohary, A., & Alwasel, I. A. (2009). The chaos and optimal control of cancer model with complete unknown parameters. *Chaos, Solitons & Fractals*, **42**(5), 2865–2874.
6. Ma, J. H., & Chen, Y. S. (2001). Study for the bifurcation topological structure and the global complicated character of a kind of nonlinear finance system(I). *Applied Mathematics and Mechanics*, **22**(11), 1240–1251.
7. Pecora, L. M., & Carroll, T. L. (1990). Synchronization in chaotic systems. *Physical Review Letters*, **64**(8), 821–824.
8. Bai, E. W., & Lonngren, K. E. (1997). Synchronization of two Lorenz systems using active control. *Chaos, Solitons & Fractals*, **8**(1), 51–58.
9. Toopchi, Y., & Wang, J. D. (2014). Chaos control and synchronization of a hyperchaotic Zhou system by integral sliding mode control. *Entropy*, **16**(12), 6539–6552.
10. Wang, F., & Liu, C. (2007). Synchronization of unified chaotic system based on passive control. *Physica D*, **225**(1), 55–60.
11. Yu, S. H., Hyun, C. H., & Park, M. (2011). Backstepping control and synchronization for 4-D Lorenz-Stenflo chaotic system with single input. *International Journal of Fuzzy Logic and Intelligent Systems*, **11**(3), 143–148.

12. Yang, S. J., Li, C. D., & Huang, T. W. (2014). Impulsive control and synchronization of memristor-based chaotic circuits. *International Journal of Bifurcation and Chaos*, **24**(12), 1450162.
13. Meijboom, B., Schmidt-Bakx, S., & Westert, G. (2011). Supply chain management practices for improving patient-oriented care. *Supply Chain Management – An International Journal*, **16**(3), 166–175.
14. Lee, L. H., Padmanabhan, V., & Whang, S. (1997). Information distortion in a supply chain: The bullwhip effect. *Management Science*, **43**(4), 546–558.
15. Ohdar, R., & Ray, P. K. (2004). Performance measurement and evaluation of suppliers in supply chain: an evolutionary fuzzy-based approach. *Journal of Manufacturing Technology Management*, **15**(8), 723–734.
16. Hwarng, H. B., & Xie, N. (2008). Understanding supply chain dynamics: A chaos perspective. *European Journal of Operational Research*, **184**(3), 1163–1178.
17. Wu, Y., & Zhang, D. Z. (2007). Demand fluctuation and chaotic behaviour by interaction between customers and suppliers. *International Journal of Production Economics*, **107**(1), 250–259.
18. Yao, M. J., & Hsu, H. W. (2009). A new spanning tree-based genetic algorithm for the design of multi-stage supply chain networks with nonlinear transportation costs. *Optimization and Engineering*, **10**(2), 219–237.
19. Fawcett, S. E., & Waller, M. A. (2011). Making sense out of chaos: Why theory is relevant to supply chain research. *Journal of Business Logistics*, **32**(1), 1–5.
20. Ramirez, S. A., & Pena, G. (2011). Analysis of chaotic behaviour in supply chain variables. *Journal of Economics, Finance & Administrative Science*, **16**(31), 85–106.
21. Jia, J. D. (2013). Dynamical behaviors of a nonlinear production-inventory supply chain system. *25th Chinese Control and Decision Conference (CCDC)*, IEEE, Guiyang, China, May 25–27, pp. 805–810.
22. Hwarng, H. B., & Yuan, X. C. (2014). Interpreting supply chain dynamics: A quasi-chaos perspective. *European Journal of Operational Research*, **233**(3), 566–579.
23. Zhang, L., Li, Y. J., & Xu, Y. Q. (2006). Chaos synchronization of bullwhip effect in a supply chain. *13th International Conference on Management Science and Engineering (ICMSE'06)*, IEEE, Lille, France, October 05–07, pp. 557–560.
24. Anne, K. R., Chedjou, J. C., & Kyamakya, K. (2009). Bifurcation analysis and synchronisation issues in a three-echelon supply chain. *International Journal of Logistics: Research and Applications*, **12**(5), 347–362.
25. Goksu, A., Kocamaz, U. E., & Uyaroglu, Y. (2015). Synchronization and control of chaos in supply chain management. *Computers & Industrial Engineering*, **86**, 107–115.
26. Kocamaz, U. E., Taskin, H., Uyaroglu, Y., & Goksu, A. (2014). Synchronization of chaotic supply chains with adaptive neuro-fuzzy inference system. *Joint International Symposium on 44th International Conference on Computers & Industrial Engineering (CIE'44) and 9th International Symposium on Intelligent Manufacturing and Service Systems (IMSS'14)*, Istanbul, Turkey, October 14–16, pp. 163–174.