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THE PERFORMANCE OF THE DOUBLE SAMPLING $\bar{X}$ CHART WITH ESTIMATED PARAMETERS FOR SKEWED DISTRIBUTIONS

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The double sampling (DS) $\bar{X}$ chart is favourable among practitioners in the context of Statistical Process Control. However, in the construction of the DS $\bar{X}$ chart, process parameters are rarely known and need to be estimated from the Phase-I historical dataset. The DS $\bar{X}$ chart with estimated parameters is effective in detecting small and moderate mean shifts. Traditionally, the DS $\bar{X}$ chart with estimated parameters is designed based on the assumption of a normal underlying distribution. In many real applications, the normality assumption may not be true. Therefore, this paper aims at investigating the performance of the DS $\bar{X}$ chart with estimated parameters for skewed populations. The skewed distributions considered in this paper are the Weibull, lognormal and gamma distributions. By applying the Monte-Carlo-simulation approach, the performance of the DS $\bar{X}$ chart, in terms of the average run length and the standard deviation of the run length, are studied for different levels of skewness and various magnitudes of mean shifts. The results reveal that the performance of the DS $\bar{X}$ chart with known and estimated parameters is significantly affected by skewed distributions. Particularly, the DS $\bar{X}$ chart with small number of Phase-I samples and sample size is seriously influenced by skewed distributions when the process mean is slightly out-of-control (small shift in the process mean). At least 80 Phase-I samples are required for the DS $\bar{X}$ chart with estimated parameters to behave similarly to its known-parameter counterpart when the underlying distribution is not normal.

Keywords: Average run length; Double sampling $\bar{X}$ chart; Parameter estimation; Skewed distribution; Standard deviation of the run length; Statistical Process Control.

Introduction

Statistical Process Control (SPC) is a collection of statistical tools to monitor and improve the quality of products in manufacturing and business processes. Control chart is one of the most useful tools in SPC. The double sampling (DS) $\bar{X}$ chart proposed by Daudin (1992) is a two-stage Shewhart $\bar{X}$ chart. It is effective in detecting small and moderate process mean shifts, without increasing the sample size. The DS $\bar{X}$ chart is also an adaptive control chart as it allows the sample size to vary. The second sample is taken only if the chart's statistic of the first sample falls in the warning regions of the DS $\bar{X}$ chart's first-sample stage.

Costa (1994) and Daudin (1992) showed that the DS $\bar{X}$ chart has some advantages over the Shewhart, variable sample size (VSS), variable sampling interval (VSI), cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) $\bar{X}$ charts. Furthermore, the total sample size of the DS plan is small when the incoming quality is either poor or excellent. This is due to the rejection or acceptance of the lot on the first sample (Gupta & Walker, 2007; Ledolter & Burrill, 1999). Thus, this sampling plan leads to cost saving and high quality products. He and Grigoryan (2002) also claimed that the DS chart is a favorable choice when protection against large shifts and greater efficiency for small shifts are both important. In view of the attractiveness of the DS chart, there is a rich literature evolving around the DS-type charts. He *et al.* (2002) designed the DS and triple sampling (TS) $\bar{X}$ charts by using genetic algorithms. Carot *et al*. (2002) proposed the DSVSI $\bar{X}$ chart, which is a combination of the DS and VSI $\bar{X}$ charts. When the correlation levels within a subgroup are small and moderate, Costa and Claro (2008) found that the DS $\bar{X}$ chart outperforms the Shewhart $\bar{X}$ and VSS $\bar{X}$ charts in detecting process mean shifts. Khoo *et al*. (2011) developed the synthetic DS $\bar{X}$ chart, which generally surpasses the DS $\bar{X}$, synthetic $\bar{X}$ and EWMA $\bar{X}$ charts, though the EWMA $\bar{X}$ chart has the best sensitivity towards small process mean shifts. Teoh *et al.* (2013) suggested the optimization algorithm for the median-run-length-based DS $\bar{X}$ charts by minimizing the average sample size. The DS feature is also extended to the dispersion-type control charts. He and Grigoryan (2002) advocated the DS *S* chart to monitor the process variation in agile manufacturing. Lee *et al*. (2012) proposed the DSVSI *S* chart in order to improve the efficiency in detecting small standard deviation shifts. Recently, Chong *et al.* (2014) proposed the synthetic DS *np* chart for attributes. Compared to the standard *np*, VSS *np*, DS *np*, synthetic *np*, combined synthetic and *np* (Syn-*np*), CUSUM *np* and EWMA *np* charts, the synthetic DS *np* chart performs reasonably well in detecting increases in most of the fraction non-conforming items *p*. Owing to the attractiveness of the DS chart, the DS procedure is considered in this paper.

In real life application, SPC consists of two phases, i.e. Phase-I and Phase-II. The Phase-I control charts are used to determine the stability of a process from a historical data; while the Phase-II control charts are used to identify the occurrence of the out-of-control condition after an unknown time point. In practice, the process parameters are rarely known with certainty; thus, they are estimated from an in-control Phase-II data. However, the Phase-II control charts are seriously affected by parameter-estimation errors. This is because of the increased variability obtained from the Phase-I analysis. Therefore, the DS $\bar{X}$ chart with estimated parameters (see Khoo *et al*., 2013; Teoh *et al*., 2014), specially designed for the Phase-I samples and sample sizes, is adopted in this paper. Jensen *et al.* (2006) and Psarakis *et al.* (2014) extensively discussed the impacts of parameter estimations on various control charts. Many researchers focus on the $\bar{X}$-type control charts. These researches include the development of the Shewhart $\bar{X}$ (Chakraborti, 2007), the synthetic $\bar{X}$ (Zhang *et al*., 2011), the VSI $\bar{X}$ (Zhang *et al*., 2012), the VSS $\bar{X}$ (Castagliola *et al*., 2012) and the DS $\bar{X}$ (Khoo *et al*., 2013; Teoh *et al*., 2014) charts with estimated parameters. Regarding the dispersion-type control charts, Castagliola *et al*. (2009) proposed the *R*, *S* and S^2 charts with estimated parameters. Guo *et al*. (2015) suggested a new optimal design and chart's parameters for the synthetic S^2 chart with estimated parameters.

To date, the design of the DS $\bar{X}$ chart with estimated parameters is based on the assumption that the underlying process is normally distributed. This assumption is violated in many real applications. For example, in the context of reliability engineering, the time to failure for structural elements in aircraft and automobiles, mechanical components, as well as electronic devices are modeled by a Weibull distribution (Montgomery, 2009). In the medical area, the lifespan of a medical laser used in ophthalmic surgery follows a lognormal distribution. Moreover, gamma distribution is adopted in numerous fields, such as finance, environmental science, climatology and engineering to analyze data with right-skewed distribution (Bhaumik *et al*., 2009). For such a skewed distribution, there will be an increase in false alarm rate of a control chart designed under the normality assumption (Chang & Bai, 2001). This is because of the inconsistency of the variability pattern between the normal and asymmetric distributions. If this occurs, practitioners may conclude that the SPC is a failure as they will lose confidence in using the control charts in process monitoring. Also, time and cost will be wasted in unnecessary adjustments. Therefore, many researchers contributed to the area of control charts for skewed populations (see Chen &

Cheng, 2007; Khoo & Kassim, 2008; Torng & Lee, 2008). For the DS-type chart with known parameters, Torng and Lee (2009) and Torng *et al*. (2010) discussed the DS $\bar{X}$ and DSVSI $\bar{X}$ charts, respectively, under non-normality. Torng and Lee (2009) concluded that the DS $\bar{X}$ chart is as good as the variable parameters (VP) $\bar{X}$ chart and the former outperforms the Shewhart $\bar{X}$ chart towards small mean shifts. Torng *et al*. (2010) claimed that the DSVSI $\bar{X}$ chart has the best overall performance for monitoring small mean shifts compared to the Shewhart $\bar{X}$ and VP $\bar{X}$ charts.

This paper is structured as follows. The next section briefly explains an overview of the DS $\bar{X}$ chart. The run-length properties (average run length, ARL, standard deviation of the run length, SDRL and average sample size, ASS) of the DS $\bar{X}$ chart with known and estimated parameters are also given. It is followed by some descriptions on the statistical properties of the Weibull, lognormal and gamma distributions. Subsequently, the in-control and out-of-control performances of the DS $\bar{X}$ chart with known and estimated parameters are investigated under the Weibull, lognormal and gamma distributions. Concluding remarks are provided in the last section.

The Double Sampling $\bar{X}$ Chart

Let us assume that the quality characteristic Y, collected from a Phase-II process, follows a normal $N\left(\mu_0,\ \sigma_0^2\right)$ distribution. Here, μ_0 and σ_0^2 are the in-control mean and variance, respectively. The DS $\bar{X}$ chart can be divided into several regions, i.e. $I_1 = \left[-L_1,\ L_1\right]$, $I_2 = \left[-L,\ -L_1\right) \cup \left(L_1,\ L\right]$, $I_3 = \left(-\infty,\ -L\right) \cup \left(L,\ +\infty\right)$ and $I_4 = \left[-L_2,\ L_2\right]$. Here, $L_1 > 0$ and $L \geq L_1$ are the warning and control limits in the first-sample stage; while $L_2 > 0$ is the control limit in the combined-sample stage. With the aid of Figure 1, the operation of the DS $\bar{X}$ chart is described as follows:

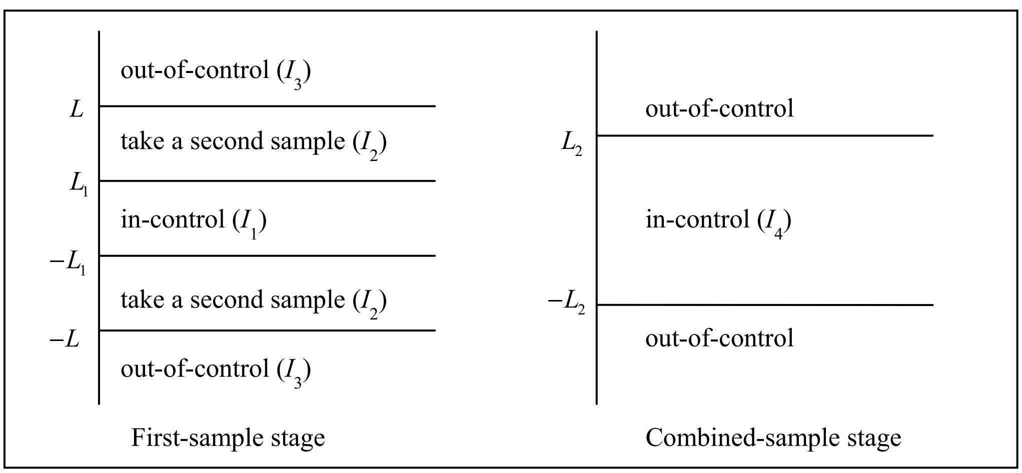

Figure 1. Graphical representation of the double sampling $\bar{X}$ chart's operation

(1) At the i^{th} sampling time, take a first sample of size n_1 from the Phase-II process. Then compute the sample mean $\bar{Y}_{1i} = \sum_{j=1}^{n_1} Y_{1i,j} / n_1$, where $Y_{1i,j}$, for $j = 1,\ 2,\ \ldots,\ n_1$, are the first-sample observations.
(2) Declare the process as in-control if $Z_{1i} = \left[\left(\bar{Y}_{1i} - \mu_0\right)\sqrt{n_1}\right] / \sigma_0 \in I_1$.
(3) Declare the process as out-of-control if $Z_{1i} \in I_3$.

(4) Take a second sample of size n_2 if $Z_{1i} \in I_2$. Then calculate the second sample mean $\bar{Y}_{2i} = \sum_{j=1}^{n_2} Y_{2i,j} / n_2$, where $Y_{2i,j}$, for $j = 1, 2, ..., n_2$, are the second-sample observations.

(5) Compute the combined-sample mean $\bar{Y}_i = \left(n_1\bar{Y}_{1i} + n_2\bar{Y}_{2i}\right) / \left(n_1 + n_2\right)$.

(6) Declare the process as in-control if $Z_i = \left[\left(\bar{Y}_i - \mu_0\right)\sqrt{n_1 + n_2}\right] / \sigma_0 \in I_4$; otherwise declare the process as out-of-control.

The Run-length Properties of the Double Sampling $\bar{X}$ Chart with Known and Estimated Parameters

For the DS $\bar{X}$ chart with known parameters, Daudin (1992) demonstrated that the ARL, SDRL and ASS can be computed as follows:

$$\text{ARL} = \frac{1}{1 - P_a}, \tag{1}$$

$$\text{SDRL} = \frac{\sqrt{P_a}}{1 - P_a} \tag{2}$$

and

$$\text{ASS} = n_1 + n_2\left[\Phi\left(L + \delta\sqrt{n_1}\right) - \Phi\left(L_1 + \delta\sqrt{n_1}\right) + \Phi\left(-L_1 + \delta\sqrt{n_1}\right) - \Phi\left(-L + \delta\sqrt{n_1}\right)\right], \tag{3}$$

respectively, where $P_a = P_{a1} + P_{a2}$ is the probability of in-control process, $\delta = \left|\mu_1 - \mu_0\right| / \sigma_0$ is the magnitude of the standardized mean shift with the out-of-control mean, μ_1 and $\Phi(\cdot)$ is the standard normal cumulative distribution function (cdf). Note that P_{a1} and P_{a2} are the probabilities that the process is deemed as in-control "by the first sample" and "after observing the second sample", respectively.

The probabilities P_{a1} and P_{a2} are equal to

$$P_{a1} = \Phi\left(L_1 + \delta\sqrt{n_1}\right) - \Phi\left(-L_1 + \delta\sqrt{n_1}\right) \tag{4}$$

and

$$P_{a2} = \int_{z \in I_2^*} \left[\Phi\left(cL_2 + rc\delta - \sqrt{\frac{n_1}{n_2}}z\right) - \Phi\left(-cL_2 + rc\delta - \sqrt{\frac{n_1}{n_2}}z\right)\right]\phi(z)\,dz, \tag{5}$$

respectively, where $\phi(\cdot)$ is the standard normal probability density function (pdf), $I_2^* = \left[-L + \delta\sqrt{n_1},\ -L_1 + \delta\sqrt{n_1}\right) \cup \left(L_1 + \delta\sqrt{n_1},\ L + \delta\sqrt{n_1}\right]$, $r = \sqrt{n_1 + n_2}$ and $c = r/\sqrt{n_2}$.

The DS $\bar{X}$ chart with estimated parameters proposed by Khoo *et al.* (2013) is implemented when both the μ_0 and σ_0 are unknown. In such a situation, these unknown parameters are estimated from an in-control Phase-I dataset consisting of i = 1, 2, …, m subgroups, each having n observations. The estimators $\hat{\mu}_0$ and $\hat{\sigma}_0$ of μ_0 and σ_0 are

$$\hat{\mu}_0 = \frac{1}{m}\sum_{i=1}^{m}\bar{X}_i \tag{6}$$

and

$$\hat{\sigma}_0 = \sqrt{\frac{1}{m(n-1)}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(X_{i,j} - \bar{X}_i\right)^2}, \tag{7}$$

respectively, where $\bar{X}_i = \sum_{j=1}^{n} X_{i,j} / n$ is the i^{th} sample mean in the Phase-I process.

Khoo *et al.* (2013) showed that the ARL, SDRL and ASS for the DS $\bar{X}$ chart with estimated parameters are calculated as

$$\text{ARL} = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} \frac{1}{1-\hat{P}_a} f_U(u) f_V(v) \, dv du, \tag{8}$$

$$\text{SDRL} = \sqrt{\int_{-\infty}^{+\infty} \int_{0}^{+\infty} \frac{1+\hat{P}_a}{\left(1-\hat{P}_a\right)^2} f_U(u) f_V(v) \, dv du - \text{ARL}^2} \tag{9}$$

and

$$\text{ASS} = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} \left(n_1 + n_2 \hat{P}_2\right) f_U(u) f_V(v) \, dv du, \tag{10}$$

respectively, where $\hat{P}_a = \hat{P}_{a1} + \hat{P}_{a2}$ is the conditional probability of the in-control process and the conditional probability $\hat{P}_2$ in Equation (10) is equal to

$$\hat{P}_2 = \Phi\left[U\sqrt{\frac{n_1}{mn}} - VL_1 - \delta\sqrt{n_1}\right] - \Phi\left[U\sqrt{\frac{n_1}{mn}} - VL - \delta\sqrt{n_1}\right] + \Phi\left[U\sqrt{\frac{n_1}{mn}} + VL - \delta\sqrt{n_1}\right] - \Phi\left[U\sqrt{\frac{n_1}{mn}} + VL_1 - \delta\sqrt{n_1}\right]. \tag{11}$$

Note that the conditional probabilities $\hat{P}_{a1}$ and $\hat{P}_{a2}$ are

$$\hat{P}_{a1} = \Phi\left[U\sqrt{\frac{n_1}{mn}} + VL_1 - \delta\sqrt{n_1}\right] - \Phi\left[U\sqrt{\frac{n_1}{mn}} - VL_1 - \delta\sqrt{n_1}\right] \tag{12}$$

and

$$\hat{P}_{a2} = \int_{z \in I_2} \hat{P}_4 \, V\phi\left(U\sqrt{\frac{n_1}{mn}} + Vz - \delta\sqrt{n_1}\right) dz, \tag{13}$$

respectively, where the conditional probability $\hat{P}_4$ is obtained as

$$\hat{P}_4 = \Phi\left[U\sqrt{\frac{n_2}{mn}} + V\left(\frac{L_2\sqrt{n_1+n_2} - z\sqrt{n_1}}{\sqrt{n_2}}\right) - \delta\sqrt{n_2}\right] - \Phi\left[U\sqrt{\frac{n_2}{mn}} - V\left(\frac{L_2\sqrt{n_1+n_2} + z\sqrt{n_1}}{\sqrt{n_2}}\right) - \delta\sqrt{n_2}\right]. \tag{14}$$

The random variables U and V in Equations (11) to (14) are defined as $U = \left(\hat{\mu}_0 - \mu_0\right) \times \left(\sqrt{mn}/\sigma_0\right)$ and $V = \hat{\sigma}_0/\sigma_0$, respectively. The pdfs of U, $f_U(u)$ and V, $f_V(v)$ in Equations (8) to (10) are

$$f_U(u) = \phi(u) \tag{15}$$

and

$$f_V(v) = 2v \, f_\gamma\left(v^2 \middle| \frac{m(n-1)}{2}, \frac{2}{m(n-1)}\right), \tag{16}$$

respectively, where $f_\gamma(\cdot)$ is the pdf of the gamma distribution.

Statistical Properties of the Weibull, Lognormal and Gamma Distributions

In this paper, we investigate the performances of the DS $\bar{X}$ chart with known and estimated parameters under Weibull, lognormal and gamma distributions. These three skewed distributions are considered in this paper because of their high flexibility. With the appropriate selection of parameters, these distributions are able to represent a wide range of skewness, ranging from almost symmetric to highly skewed.

The cdf of the Weibull distribution is (Bai & Choi, 1995)

$$F(w) = 1 - e^{-(\lambda w)^{\beta}}, \text{ for } w \geq 0, \tag{17}$$

where $\lambda > 0$ and $\beta > 0$ are the scale and shape parameters, respectively. The skewness (γ) is computed by (Prabhakar *et al.*, 2004)

$$\gamma = \frac{\Gamma\left(1+\frac{3}{\beta}\right) - 3\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{2}{\beta}\right) + 2\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^3}{\left[\Gamma\left(1+\frac{2}{\beta}\right) - \left\{\Gamma\left(1+\frac{1}{\beta}\right)\right\}^2\right]^{\frac{3}{2}}}, \tag{18}$$

where $\Gamma(\cdot)$ is the gamma function. Since the skewness only depends on the shape parameter β, the scale parameter $\lambda = 1$ is chosen for convenience. When $\lambda = 1$, the probability $P_W = \Pr(W \leq \mu)$, in-control mean and standard deviation are obtained as (Khoo *et al.*, 2008)

$$P_W = 1 - e^{-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^{\beta}}, \tag{19}$$

$$\mu_{W,0} = \Gamma\left(1+\frac{1}{\beta}\right) \tag{20}$$

and

$$\sigma_{W,0} = \sqrt{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2}, \tag{21}$$

respectively, where μ is the target mean value of *W*.

The cdf of a lognormal distribution is (Bai & Choi, 1995)

$$F(w) = \Phi\left(\frac{\ln w - \theta}{\sigma_{\text{LN}}}\right), \text{ for } w > 0, \tag{22}$$

where θ and σ_{LN} are the location and scale parameters, respectively. The skewness (γ) is calculated by (Blackwood, 1992)

$$\gamma = \left(e^{\sigma_{\text{LN}}^2} + 2\right)\sqrt{e^{\sigma_{\text{LN}}^2} - 1}. \tag{23}$$

Note that the location parameter $\theta = 0$ is selected as the skewness does not depend on this parameter. When $\theta = 0$, the probability $P_W = \Pr(W \leq \mu)$, in-control mean and standard deviation are obtained as (Khoo *et al.*, 2008)

$$P_W = \Phi\left(\frac{\sigma_{\text{LN}}}{2}\right), \tag{24}$$

$$\mu_{W,0} = e^{\frac{1}{2}\sigma_{\text{LN}}^2} \tag{25}$$

and

$$\sigma_{W,0} = \sqrt{e^{\sigma_{\text{LN}}^2}\left(e^{\sigma_{\text{LN}}^2} - 1\right)}, \tag{26}$$

respectively.

For a gamma distribution with location parameter of zero and scale parameter of one, its cdf is (Johnson & Kotz, 1970)

$$F\left(w\right) = \frac{\Gamma_W\left(\alpha\right)}{\Gamma\left(\alpha\right)}, \text{ for } w \geq 0,\ \alpha > 0, \tag{27}$$

where $\Gamma_W\left(\alpha\right) = \int_0^w k^{\alpha-1} e^{-k} dk$ and $\Gamma\left(\alpha\right) = \int_0^\infty k^{\alpha-1} e^{-k} dk$. The skewness $\left(\gamma\right)$ is calculated by (Bowman & Shenton, 2014)

$$\gamma = \frac{2}{\sqrt{\alpha}}, \tag{28}$$

where α is the shape parameter. Then, the probability $P_W = \Pr\left(W \leq \mu\right)$, in-control mean and standard deviation are obtained as (Khoo *et al.*, 2008)

$$P_W = F\left(\alpha\right), \tag{29}$$

$$\mu_{W,0} = \alpha \tag{30}$$

and

$$\sigma_{W,0} = \sqrt{\alpha}, \tag{31}$$

respectively.

Performance Studies

For the sake of comparison, besides the Weibull, lognormal and gamma distributions, this paper also considers the normal distribution because the DS $\bar{X}$ chart with known and estimated parameters is designed under normal distribution. When δ_{opt} = 1.0, the optimal chart's parameters (n_1, n_2, L_1, L, L_2) for the DS $\bar{X}$ chart with known ($m = +\infty$) and estimated ($m \in$ {10, 20, 40, 80}) parameters listed in Table 1 are obtained under normal distribution. Here, δ_{opt} represents the desired mean shift for which a quick detection is required. The optimization algorithm and the related formulae can be found in Khoo *et al.* (2013) and Equations (1) to (16), respectively. Note that all the optimal chart's parameters (n_1, n_2, L_1, L, L_2) in Table 1 must attain ARL$_0$ = 250 and ASS$_0$ = $n \in$ {5, 10}. Note that the subscripts "0" and "1" for ARL, SDRL and ASS refer to the in-control and out-of-control cases, respectively.

Table 1. Optimal chart's parameters (n_1, n_2, L_1, L, L_2) of the DS $\bar{X}$ chart with known ($m = +\infty$) and estimated ($m \in$ {10, 20, 40, 80}) parameters when ARL$_0$ = 250, ASS$_0$ = $n \in$ {5, 10} and δ_{opt} = 1.0.

n	$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
	(n_1, n_2, L_1, L, L_2)	(n_1, n_2, L_1, L, L_2)	(n_1, n_2, L_1, L, L_2)	(n_1, n_2, L_1, L, L_2)	(n_1, n_2, L_1, L, L_2)
5	(3, 11, 1.398, 4.108, 2.672)	(3, 11, 1.367, 5.006, 2.698)	(3, 11, 1.351, 5.446, 2.696)	(3, 11, 1.343, 5.378, 2.687)	(3, 11, 1.335, 5.035, 2.665)
10	(8, 7, 1.116, 5.298, 2.907)	(8, 7, 1.092, 5.293, 2.902)	(8, 7, 1.080, 5.070, 2.890)	(8, 7, 1.074, 5.158, 2.880)	(8, 7, 1.068, 5.016, 2.865)

Tables 2 to 6 compare the in-control and out-of-control ARL and SDRL performances of the DS $\bar{X}$ chart with known ($m = +\infty$) and estimated ($m \in \{10, 20, 40, 80\}$) parameters when the underlying distributions are normal, Weibull, lognormal and gamma. The chart's parameters (n_1, n_2, L_1, L, L_2) displayed in Table 1 and the Monte-Carlo simulation programs written in Statistical Analysis Software (SAS) are used to obtain the ARLs and SDRLs of the DS $\bar{X}$ chart with known and estimated parameters. For a given skewness γ, the shape parameters β and α of the Weibull and gamma distributions, respectively, as well as the scale parameter σ_{LN} of the lognormal distribution can be uniquely determined by using Mathematica software. In this paper, $\gamma \in \{0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0\}$ are considered. When $\gamma = 0$, the distribution is symmetry. Low levels of skewness are represented by $\gamma \in \{0.5, 1.0\}$; moderate levels of skewness are represented by $\gamma \in \{1.5, 2.0\}$; while the high levels of skewness are represented by $\gamma \in \{2.5, 3.0\}$. For Weibull, lognormal and gamma distributions, a shift in the process mean is indicated by $\mu_{W,1} = \mu_{W,0} + \delta\sigma_{W,0}$. When $\delta = 0$, the process is in-control; otherwise, the process is out-of-control.

Tables 2 and 3 present the ARL_0 and SDRL_0 values of the DS $\bar{X}$ chart with known and estimated parameters when $n \in \{5, 10\}$, respectively. Since the DS $\bar{X}$ chart with known and estimated parameters is designed under the normal distribution, all the ARL_0 values are equal to 250. When $\gamma = 0$, the data is symmetrically distributed; thus, all the ARL_0 values for the Weibull, lognormal and gamma distributions approach that of the normal distribution. As expected, for each γ value, the differences between the (ARL_0, SDRL_0) values decrease as m increases. There is an obvious trend for the DS $\bar{X}$ chart with known parameters. From Tables 2 and 3, the ARL_0 and SDRL_0 values for the known-parameter case decrease as γ increases. This indicates that when the underlying distribution is highly skewed, implementing the DS $\bar{X}$ chart with known parameters will lead to a high false alarm rate. This situation is significantly undesirable. For small number of Phase-I samples, m, we observe that the ARL_0 values are very unstable as the SDRL_0 values are quite large, suggesting a large variation in the run-length distribution. This contributes to the unobvious trend for the DS $\bar{X}$ chart with estimated parameters. For most estimated-parameter cases, the ARL_0 and SDRL_0 values for the Weibull, lognormal and gamma distributions are larger than that of the normal distribution and its known-parameter counterpart. For example, when $m = 20$, $n = 5$ and $\gamma = 2.5$, the (ARL_0, SDRL_0) values are (508.73, 7528.93) for the Weibull-distribution case as opposed to (250.00, 406.23) for the normal-distribution case (see Table 2). Also, these (ARL_0 = 508.73, SDRL_0 = 7528.93) values decrease to (ARL_0 = 152.47, SDRL_0 = 151.85) when $m = +\infty$ (see Table 2). This single example shows that the DS $\bar{X}$ chart is remarkably affected by both the levels of skewness and parameter estimations. It is obvious that from Tables 2 and 3, all the ARL_0 values computed under the skewed distribution are very far away from the desired value, i.e. $\text{ARL}_0 = 250$.

Table 2. The (ARL_0, SDRL_0) values of the DS $\bar{X}$ chart with known ($m = +\infty$) and estimated ($m \in \{10, 20, 40, 80\}$) parameters when $n = 5$

Parameter	Distribution	γ	$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
			(ARL_0, SDRL_0)	(ARL_0, SDRL_0)	(ARL_0, SDRL_0)	(ARL_0, SDRL_0)	(ARL_0, SDRL_0)
	Normal		(250.00, 660.81)	(250.00, 406.23)	(250.00, 318.49)	(250.00, 281.10)	(250.00, 249.50)
	Weibull						
β	3.6024	0.0	(246.21, 596.03)	(249.75, 387.56)	(253.45, 315.28)	(253.34, 280.67)	(254.46, 252.12)
	2.2156	0.5	(266.84, 820.67)	(263.87, 436.83)	(258.91, 338.48)	(257.41, 292.16)	(250.43, 246.64)
	1.5639	1.0	(352.85, 1620.79)	(310.06, 711.94)	(277.15, 406.47)	(260.18, 321.79)	(232.83, 232.29)
	1.2111	1.5	(605.39, 10056.16)	(395.92, 1571.57)	(300.54, 565.90)	(256.09, 357.23)	(206.34, 206.00)
	1.0000	2.0	(1113.55, 91652.94)	(476.01, 3007.19)	(298.50, 687.33)	(232.45, 349.48)	(176.56, 175.35)
	0.8632	2.5	(692.82, 15949.59)	(508.73, 7528.93)	(272.95, 718.97)	(206.18, 323.72)	(152.47, 151.85)
	0.7686	3.0	(716.62, 79920.60)	(451.05, 14287.50)	(248.22, 1181.12)	(183.47, 292.19)	(134.83, 133.52)

	Lognormal						
	0.0003	0.0	(249.27, 641.53)	(248.11, 397.63)	(248.48, 317.39)	(248.34, 280.94)	(249.48, 249.11)
	0.1641	0.5	(276.05, 981.15)	(260.03, 452.59)	(254.44, 341.75)	(251.38, 291.14)	(243.23, 244.23)
	0.3143	1.0	(641.31, 61313.63)	(314.17, 1828.88)	(270.44, 418.62)	(250.11, 314.08)	(224.94, 224.55)
σ_{LN}	0.4435	1.5	(679.15, 22279.91)	(429.98, 5857.51)	(288.05, 731.28)	(241.91, 341.61)	(198.15, 197.91)
	0.5514	2.0	(659.69, 18131.66)	(518.54, 16046.70)	(290.76, 1246.94)	(225.95, 377.39)	(198.15, 197.91)
	0.6409	2.5	(461.73, 9884.42)	(450.85, 9323.02)	(276.78, 1527.91)	(211.22, 1019.90)	(155.15, 155.19)
	0.7156	3.0	(410.60, 11874.45)	(726.24, 100410.15)	(263.67, 1910.79)	(195.92, 651.22)	(142.16, 143.05)
	Gamma						
	40000	0.0	(253.91, 645.04)	(248.77, 395.36)	(251.66, 321.97)	(250.49, 281.36)	(251.64, 250.82)
	16.0000	0.5	(273.19, 874.63)	(260.76, 475.23)	(256.06, 336.20)	(251.61, 288.72)	(246.78, 246.39)
	4.0000	1.0	(373.71, 2684.61)	(303.98, 747.47)	(276.65, 436.41)	(257.07, 320.75)	(230.08, 229.42)
α	1.7778	1.5	(685.64, 33291.26)	(395.34, 2080.44)	(296.13, 555.59)	(251.75, 351.62)	(205.86, 175.35)
	1.0000	2.0	(1113.55, 91652.94)	(476.01, 3007.19)	(298.50, 687.33)	(232.45, 359.48)	(176.56, 175.35)
	0.6400	2.5	(960.13, 52791.86)	(473.31, 3841.95)	(275.79, 750.71)	(206.87, 330.37)	(152.89, 151.74)
	0.4444	3.0	(592.19, 10082.29)	(416.96, 3777.39)	(242.88, 620.45)	(182.02, 289.56)	(133.13, 132.68)

Table 3. The (ARL_0, SDRL_0) values of the DS $\bar{X}$ chart with known ($m=+\infty$) and estimated ($m \in \{10, 20, 40, 80\}$) parameters when $n=10$

Parameter	Distribution		$m=10$	$m=20$	$m=40$	$m=80$	$m=+\infty$
		γ	(ARL_0, SDRL_0)	(ARL_0, SDRL_0)	(ARL_0, SDRL_0)	(ARL_0, SDRL_0)	(ARL_0, SDRL_0)
	Normal		(250.00, 426.50)	(250.00, 326.81)	(250.00, 284.74)	(250.00, 265.81)	(250.00, 249.50)
	Weibull						
	3.6024	0.0	(250.22, 408.05)	(254.61, 329.36)	(256.52, 287.18)	(255.72, 269.74)	(258.21, 257.19)
	2.2156	0.5	(268.77, 484.96)	(262.23, 357.25)	(260.13, 303.35)	(257.88, 278.08)	(249.15, 247.23)
	1.5639	1.0	(324.01, 783.08)	(287.84, 465.16)	(261.80, 337.47)	(244.25, 280.68)	(224.51, 223.44)
β	1.2111	1.5	(451.56, 2993.29)	(309.91, 682.44)	(248.68, 372.70)	(219.86, 271.60)	(190.54, 190.82)
	1.0000	2.0	(581.30, 5111.70)	(303.84, 956.38)	(219.48, 360.17)	(188.85, 241.73)	(159.37, 159.17)
	0.8632	2.5	(639.38, 7230.24)	(270.09, 832.70)	(189.80, 323.05)	(161.93, 208.80)	(136.61, 136.12)
	0.7686	3.0	(713.60, 17147.59)	(245.71, 1111.82)	(169.98, 549.01)	(142.06, 188.31)	(120.35, 119.82)
	Lognormal						
	0.0003	0.0	(248.75, 421.04)	(249.61, 322.90)	(250.23, 285.29)	(249.87, 266.53)	(248.81, 247.43)
	0.1641	0.5	(268.13, 513.97)	(258.25, 359.61)	(251.02, 294.44)	(246.08, 267.05)	(238.56, 239.64)
	0.3143	1.0	(334.46, 1494.60)	(273.01, 466.51)	(246.72, 328.89)	(231.73, 266.58)	(212.92, 213.72)
σ_{LN}	0.4435	1.5	(479.69, 5040.01)	(293.28, 807.43)	(233.79, 353.37)	(207.19, 254.38)	(183.22, 182.28)
	0.5514	2.0	(766.49, 28193.71)	(294.46, 1372.34)	(210.40, 366.83)	(181.86, 235.72)	(157.11, 157.22)
	0.6409	2.5	(687.92, 16708.70)	(282.08, 2122.64)	(189.08, 385.69)	(160.23, 223.92)	(137.51, 136.37)
	0.7156	3.0	(650.70, 17344.70)	(271.07, 5535.71)	(172.23, 465.43)	(146.26, 260.35)	(123.93, 123.59)
	Gamma						
	40000	0.0	(249.78, 427.09)	(248.94, 326.58)	(249.55, 284.08)	(250.43, 264.31)	(249.94, 247.93)
	16.0000	0.5	(265.94, 493.35)	(258.71, 355.67)	(251.89, 299.98)	(245.30, 266.41)	(240.98, 239.58)
	4.0000	1.0	(326.39, 921.03)	(281.93, 456.74)	(253.90, 333.08)	(237.61, 273.86)	(220.49, 217.77)
α	1.7778	1.5	(456.23, 3582.46)	(304.46, 653.44)	(245.93, 360.17)	(216.83, 268.84)	(189.39, 190.49)
	1.0000	2.0	(581.30, 5111.70)	(303.84, 956.38)	(219.48, 360.17)	(188.85, 241.73)	(159.37, 159.17)
	0.6400	2.5	(821.03, 66699.64)	(273.02, 810.07)	(187.99, 312.32)	(160.07, 207.73)	(136.64, 135.66)
	0.4444	3.0	(621.71, 7816.82)	(242.94, 1054.29)	(165.77, 286.07)	(141.55, 186.45)	(119.74, 119.60)

Tables 4 to 6 give us an overview of the (ARL_1, $SDRL_1$) values of the DS $\bar{X}$ chart with known and estimated parameters when n = 5 and the underlying distributions are Weibull, lognormal and gamma, respectively. For the known-parameter case, the (ARL_1, $SDRL_1$) values increase as γ increases for moderate shifts, i.e. $\delta \in$ {0.50, 0.75, 1.00}, Weibull and gamma distributions. However, for these two distributions, the (ARL_1, $SDRL_1$) values decrease as γ increases for small and large shifts, i.e. $\delta \in$ {0.25, 1.50, 2.00}. For lognormal distributions, the (ARL_1, $SDRL_1$) values decrease as γ increases for all levels of mean shifts. The performances of the DS $\bar{X}$ chart with estimated parameters are very different compared to its known-parameter counterpart. For Weibull and gamma distributions, the (ARL_1, $SDRL_1$) values for most of the estimated-parameter cases increase as γ increases. For example, when $m = 20$, $n = 5$ and $\delta = 0.25$, the (ARL_1, $SDRL_1$) values for γ = 0.0 is (77.92, 159.42) for the Weibull-distribution case; while that for γ = 3.0, the (ARL_1, $SDRL_1$) values increase to (167.75, 9237.16) (see Table 4). This single example indicates that the sensitivity of the DS $\bar{X}$ chart with estimated parameters decreases tremendously when γ increases. For lognormal distribution, the ARL_1 values for most of the estimated-parameter cases decrease as γ increases; while the $SDRL_1$ values for small δ and m increase as γ increases.

It is clear that from Tables 4 to 6, for all the three non-normal distributions, the (ARL_1, $SDRL_1$) values for the estimated-parameter case converge towards its known-parameter counterpart for fixed values of γ and δ. For a fixed γ, the differences of the (ARL_1, $SDRL_1$) values between the estimated- and known-parameter cases decrease as m or δ increases. For small and moderate shifts ($\delta \le 1.00$), at least 80 Phase-I samples are required for the DS $\bar{X}$ chart with estimated parameters to behave like its known-parameter counterpart. For large shifts $(\delta \ge 1.50)$, the (ARL_1, $SDRL_1$) values of the estimated-parameter case are almost the same as those of the known-parameter case, regardless of the values of m and γ used.

Table 4. The (ARL_1, $SDRL_1$) values of the DS $\bar{X}$ chart with known ($m = +\infty$) and estimated ($m \in$ {10, 20, 40, 80}) parameters when $n = 5$ and the underlying distribution is Weibull.

			$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
β	γ	δ	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)
3.6024	0.0	0.25	(104.58, 325.51)	(77.92, 159.42)	(60.37, 89.80)	(51.97, 64.73)	(44.13, 43.72)
		0.50	(16.23, 46.80)	(11.56, 19.23)	(9.77, 11.79)	(8.99, 9.56)	(8,13, 7.62)
		0.75	(3.95,6.23)	(3.36, 3.55)	(3.12, 2.85)	(3.01, 2.59)	(2.87, 2.33)
		1.00	(1.89, 1.65)	(1.78, 1.29)	(1.72, 1.16)	(1.68, 1.10)	(1.65, 1.04)
		1.50	(1.15, 0.44)	(1.13, 0.40)	(1.13, 0.38)	(1.12, 0.37)	(1.12, 0.36)
		2.00	(1.02, 0.16)	(1.02, 0.14)	(1.02, 0.14)	(1.02, 1.13)	(1.02, 0.13)
2.2156	0.5	0.25	(126.31, 693.19)	(80.58, 197.60)	(58.70, 97.26)	(49.41, 63.39)	(41.33, 40.83)
		0.50	(20.16, 78.09)	(12.83, 25.19)	(10.36, 13.21)	(9.41, 10.35)	(8.38, 7.84)
		0.75	(4.62, 9.70)	(3.69, 4.49)	(3.34, 3.26)	(3.18, 2.82)	(3.01, 2.45)
		1.00	(2.05, 2.12)	(1.85, 1.46)	(1.77, 1.25)	(1.73, 1.15)	(1.70, 1.08)
		1.50	(1.16, 0.46)	(1.13, 0.39)	(1.12, 0.37)	(1.12, 0.36)	(1.11, 0.35)
		2.00	(1.02, 0.14)	(1.01, 0.11)	(1.01, 0.10)	(1.01, 0.09)	(1.01, 0.10)
1.5639	1.0	0.25	(164.75, 1141.85)	(85.24, 270.14)	(57.47, 102.29)	(47.40, 62.93)	(39.17, 38.55)
		0.50	(26.78, 226.50)	(14.36, 32.83)	(11.15,15.30)	(9.94, 11.31)	(8.75, 8.23)
		0.75	(5.79, 47.22)	(4.07, 6.01)	(3.61, 3.79)	(3.36, 3.11)	(3.15, 2.59)
		1.00	(2.27, 3.42)	(1.96, 1.74)	(1.83, 1.37)	(1.78, 1.23)	(1.73, 1.12)
		1.50	(1.15, 0.50)	(1.13, 0.40)	(1.11, 0.36)	(1.10, 0.34)	(1.10, 0.32)
		2.00	(1.01, 0.12)	(1.01, 0.08)	(1.00, 0.06)	(1.00, 0.05)	(1.00, 0.04)

1.2111	1.5	0.25	(300.44, 2497.00)	(94.44, 933.78)	(56.84, 106.05)	(46.47, 62.96)	(38.08, 37.59)
		0.50	(40.62, 1030.20)	(16.29, 44.98)	(12.02, 17.72)	(10.56, 12.55)	(9.13, 8.61)
		0.75	(7.96, 118.67)	(4.61, 8.09)	(3.89, 4.48)	(3.58, 3.47)	(3.29, 2.75)
		1.00	(2.62, 8.06)	(2.08, 2.11)	(1.91, 1.52)	(1.84, 1.32)	(1.77, 1.16)
		1.50	(1.17, 0.61)	(1.19, 0.41)	(1.10, 0.34)	(1.09, 0.31)	(1.08, 0.29)
		2.00	(1,01, 0.12)	(1.00, 0.05)	(1.00, 0.03)	(1.00, 0.01)	(1.00, 0.00)
1.0000	2.0	0.25	(259.13, 4983.50)	(97.31, 568.04)	(57.98, 120.72)	(46.23, 64.52)	(37.08, 36.46)
		0.50	(54.58, 1140.51)	(19.31, 80.22)	(13.15, 21.67)	(11.26, 13.97)	(9.55, 9.03)
		0.75	(11.37, 161.58)	(5.33, 13.19)	(4.21, 5.48)	(3.82, 3.88)	(3.45, 2.90)
		1.00	(3.32, 18.60)	(2.23, 2.87)	(2.00, 1.74)	(1.89, 1.43)	(1.80, 1.19)
		1.50	(1.20, 2.31)	(1.11, 0.45)	(1.08, 0.32)	(1.06, 0.27)	(1.05, 0.23)
		2.00	(1.01, 0.15)	(1.00, 0.05)	(1.00, 0.02)	(1.00, 0.00)	(1.00, 0.00)
0.8632	2.5	0.25	(322.87, 11011.58)	(119.72, 3083.42)	(59.63, 153.92)	(46.41, 68.01)	(36.65, 35.98)
		0.50	(75.26, 2063.79)	(23.45, 194.37)	(14.49, 29.02)	(12.07, 16.05)	(10.02, 9.47)
		0.75	(29.36, 2363.49)	(6.36, 25.78)	(4.62, 7.30)	(4.06, 4.59)	(3.63, 3.09)
		1.00	(21.61, 4839.12)	(2.49, 7.27)	(2.08, 2.12)	(1.94, 1.54)	(1.82, 1.22)
		1.50	(1.37, 27.58)	(1.11, 0.62)	(1.07, 0.31)	(1.05, 0.24)	(1.03, 0.16)
		2.00	(1.02, 0.35)	(1.00, 0.06)	(1.00, 0.02)	(1.00, 0.00)	(1.00, 0.00)
0.7686	3.0	0.25	(503.51, 69413.30)	(167.75, 9237.16)	(62.77, 252.61)	(47.30, 73.39)	(36.57, 36.07)
		0.50	(126.14, 8240.57)	(32.93, 743.19)	(16.20, 48.44)	(12.89, 17.92)	(10.51, 9.98)
		0.75	(73.78, 10970.62)	(8.30, 89.41)	(5.18, 23.51)	(4.34, 5.23)	(3.78, 3.23)
		1.00	(9.48, 326.03)	(2.95, 27.96)	(2.20, 2.81)	(1.99, 1.69)	(1.84, 1.24)
		1.50	(4.47, 596.07)	(1.13, 0.94)	(1.07, 0.35)	(1.03, 0.21)	(1.00, 0.05)
		2.00	(1.13, 26.90)	(1.00, 0.10)	(1.00, 0.02)	(1.00, 0.00)	(1.00, 0.00)

Table 5. The (ARL_1, $SDRL_1$) values of the DS $\bar{X}$ chart with known ($m = +\infty$) and estimated ($m \in \{10, 20, 40, 80\}$) parameters when $n = 5$ and the underlying distribution is Lognormal

			$m = 10$	$m = 20$	$m = 40$	$m = 80$	$m = +\infty$
σ_{LN}	γ	δ	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)
0.0003	0.0	0.25	(106.90, 339.62)	(78.73, 166.38)	(61.37, 93.09)	(52.15, 64.83)	(44.51, 43.95)
		0.50	(16.80, 52.24)	(11.69, 19.45)	(9.90, 11.98)	(9.03, 9.52)	(8.22, 7.66)
		0.75	(4.02, 6.82)	(3.41, 3.69)	(3.16, 2.94)	(3.02, 2.60)	(2.88, 2.31)
		1.00	(1.90, 1.72)	(1.78, 1.30)	(1.72, 1.16)	(1.68, 1.09)	(1.65, 1.04)
		1.50	(1.15, 0.43)	(1.13, 0.39)	(1.12, 0.37)	(1.12, 0.37)	(1.11, 0.36)
		2.00	(1.02, 0.16)	(1.02, 0.15)	(1.02, 0.14)	(1.02, 0.13)	(1.02, 0.13)
0.1641	0.5	0.25	(92.24, 434.08)	(61.54, 148.83)	(46.31, 70.83)	(39.64, 48.64)	(33.93, 33.46)
		0.50	(13.44, 69.45)	(9.36, 15.28)	(8.02, 9.57)	(7.35, 7.56)	(6.84, 6.31)
		0.75	(3.44, 5.33)	(2.98, 3.09)	(2.79, 2.46)	(2.69, 2.23)	(2.58, 2.01)
		1.00	(1.77, 1.48)	(1.67, 1.16)	(1.61, 1.03)	(1.60, 0.99)	(1.57, 0.95)
		1.50	(1.13, 0.40)	(1.12, 0.36)	(1.11, 0.34)	(1.10, 0.34)	(1.10, 0.33)
		2.00	(1.02, 0.14)	(1.02, 0.13)	(1.01, 0.12)	(1.01, 0.12)	(1.01, 0.12)
0.3143	1.0	0.25	(88.81, 1575.00)	(48.85, 150.09)	(35.78, 56.13)	(30.84, 37.89)	(26.42, 25.98)
		0.50	(10.31, 38.19)	(7.48, 12.27)	(6.54, 7.66)	(6.01, 6.05)	(5.63, 5.12)

		0.75	(2.96, 5.03)	(2.57, 2.50)	(2.44, 2.06)	(2.36, 1.88)	(2.28, 1.71)
		1.00	(1.61, 1.23)	(1.54, 0.98)	(1.50, 0.90)	(1.49, 0.86)	(1.46, 0.83)
		1.50	(1.10, 0.34)	(1.09, 0.31)	(1.08, 0.30)	(1.08, 0.30)	(1.08, 0.29)
		2.00	(1.01, 0.12)	(1.01, 0.10)	(1.01, 0.10)	(1.01, 0.09)	(1.01, 0.09)
0.4435	1.5	0.25	(90.98, 3665.13)	(40.93, 484.26)	(28.56, 46.21)	(24.60, 30.09)	(21.19, 20.54)
		0.50	(9.36, 124.24)	(6.09, 11.09)	(5.35, 6.30)	(5.00, 4.93)	(4.67, 4.13)
		0.75	(2.51, 4.76)	(2.24, 2.08)	(2.13, 1.69)	(2.08, 1.57)	(2.01, 1.43)
		1.00	(1.47, 1.01)	(1.42, 0.83)	(1.39, 0.76)	(1.38, 0.74)	(1.37, 0.71)
		1.50	(1.07, 0.28)	(1.06, 0.26)	(1.06, 0.25)	(1.06, 0.24)	(1.05, 0.23)
		2.00	(1.01, 0.08)	(1.01, 0.07)	(1.00, 0.07)	(1.05, 0.07)	(1.00, 0.06)
0.5514	2.0	0.25	(98.46, 6688.73)	(33.58, 180.10)	(23.79, 42.75)	(20.43, 25.44)	(17.69, 17.12)
		0.50	(8.42, 284.64)	(5.15, 14.66)	(4.50,5.18)	(4.21, 4.07)	(3.97, 3.42)
		0.75	(2.20, 5.00)	(1.97, 1.80)	(1.90, 1.42)	(1.85, 1.31)	(1.81, 1.21)
		1.00	(1.37, 1.02)	(1.32, 0.71)	(1.30, 0.65)	(1.29, 0.62)	(1.29, 0.60)
		1.50	(1.04, 0.22)	(1.04, 0.21)	(1.04, 0.20)	(1.03, 0.19)	(1.03, 0.19)
		2.00	(1.00, 0.05)	(1.00, 0.05)	(1.00, 0.04)	(1.00, 0.04)	(1.00, 0.04)
0.6409	2.5	0.25	(96.75, 7856.81)	(31.57, 588.31)	(20.60, 45.11)	(17.54, 23.33)	(15.15, 14.60)
		0.50	(6.99, 133.67)	(4.39, 14.99)	(3.80, 4.34)	(3.64, 3.53)	(3.42, 2.88)
		0.75	(2.00, 9.42)	(1.78, 1.69)	(1.71, 1.22)	(1.68, 1.10)	(1.65, 1.03)
		1.00	(1.28, 0.84)	(1.25, 0.59)	(1.23, 0.55)	(1.23, 0.53)	(1.22, 0.52)
		1.50	(1.03, 0.17)	(1.02, 0.16)	(1.02, 0.15)	(1.02, 0.14)	(1.02, 0.14)
		2.00	(1.00, 0.03)	(1.00, 0.03)	(1.00, 0.03)	(1.00, 0.03)	(1.00, 0.03)
0.7156	3.0	0.25	(77.27, 4853.79)	(28.14, 47.26)	(18.53, 94.23)	(15.44, 25.89)	(13.26, 12.74)
		0.50	(5.75, 105.76)	(4.00, 47.26)	(3.37, 4.39)	(3.15, 2.94)	(3.01, 2.46)
		0.75	(1.80, 8.25)	(1.62, 1.64)	(1.57, 1.05)	(1.55, 0.95)	(1.52, 0.89)
		1.00	(1.21, 0.92)	(1.18, 0.51)	(1.77, 0.47)	(1.17, 0.45)	(1.16, 0.44)
		1.50	(1.02, 0.13)	(1.01, 0.12)	(1.01, 0.11)	(1.01, 0.11)	(1.01, 0.10)
		2.00	(1.00, 0.02)	(1.00, 0.02)	(1.00, 0.01)	(1.00, 0.02)	(1.00, 0.01)

Table 6. The (ARL_1, $SDRL_1$) values of the DS $\bar{X}$ chart with known ($m = +\infty$) and estimated ($m \in \{10, 20, 40, 80\}$) parameters when $n = 5$ and the underlying distribution is gamma

			$m=10$	$m=20$	$m=40$	$m=80$	$m=+\infty$
α	γ	δ	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)	(ARL_1, $SDRL_1$)
40000	0.0	0.25	(107.74, 350.36)	(79.14, 170.98)	(61.61, 96.30)	(52.75, 64.98)	(44.10, 43.77)
		0.50	(17.39, 72.81)	(11.72, 20.24)	(9.88, 11.92)	(9.05, 9.58)	(8.22, 7.69)
		0.75	(4.03, 6.89)	(3.43, 3.78)	(3.14, 2.90)	(3.02, 2.61)	(2.88, 2.34)
		1.00	(1.89, 1.70)	(1.78, 1.32)	(1.71, 1.15)	(1.69, 1.10)	(1.65, 1.04)
		1.50	(1.15, 0.44)	(1.13, 0.39)	(1.12, 0.37)	(1.12, 0.37)	(1.12, 0.36)
		2.00	(1.02, 0.16)	(1.02, 0.14)	(1.02, 0.13)	(1.02, 0.14)	(1.02, 0.13)
16.0000	0.5	0.25	(131.44, 619.38)	(89.92, 212.60)	(59.29, 98.37)	(49.87, 63.33)	(41.48, 40.82)
		0.50	(22.25, 144.95)	(12.89, 25.29)	(10.62, 13.84)	(9.53, 10.49)	(8.54, 8.00)
		0.75	(4.86, 30.32)	(3.74, 4.64)	(3.35, 3.27)	(3.18, 2.82)	(3.01, 2.47)
		1.00	(2.04, 2.24)	(1.85, 1.49)	(1.78, 1.26)	(1.73, 1.15)	(1.69, 1.07)

		1.50	(1.15, 0.46)	(1.13, 0.39)	(1.12, 0.37)	(1.14, 0.36)	(1.10, 0.34)
		2.00	(1.02, 0.14)	(1.01, 0.12)	(1.01, 0.11)	(1.01, 0.10)	(1.01, 0.10)
4.0000	1.0	0.25	(186.28, 3481.75)	(86.57, 277.94)	(58.78, 110.10)	(47.90, 62.66)	(39.61, 39.43)
		0.50	(29.17, 219.66)	(14.79, 39.73)	(11.29, 15.32)	(10.06, 11.32)	(8.83, 8.32)
		0.75	(6.62, 106.95)	(4.15, 6.49)	(3.60, 3.83)	(3.38, 3.13)	(3.15, 2.62)
		1.00	(2.31, 4.26)	(1.96, 1.81)	(1.82, 1.35)	(1.77, 1.22)	(1.72, 1.11)
		1.50	(1.16, 0.51)	(1.12, 0.40)	(1.11, 0.35)	(1.10, 0.34)	(1.09, 0.32)
		2.00	(1.01, 0.12)	(1.01, 0.08)	(1.00, 0.07)	(1.00, 0.06)	(1.00, 0.06)
1.7778	1.5	0.25	(279.99, 15286.56)	(94.22, 493.84)	(58.20, 111.57)	(47.00 63.23)	(37.95, 37.40)
		0.50	(38.91, 651.55)	(16.88, 55.35)	(12.21,18.44)	(10.59,12.38)	(9.17, 8.69)
		0.75	(8.48, 101.78)	(4.70, 9.58)	(3.87, 4.46)	(3.58, 3.45)	(3.30, 2.76)
		1.00	(2.73, 9.67)	(2.08, 2.28)	(1.90, 1.51)	(1.82, 1.30)	(1.76, 1.15)
		1.50	(1.17, 0.65)	(1.11, 0.40)	(1.10, 0.34)	(1.09, 0.31)	(1.08, 0.28)
		2.00	(1.01, 0.12)	(1.00, 0.06)	(1.00, 0.03)	(1.00, 0.02)	(1.00, 0.00)
1.0000	2.0	0.25	(259.13, 4983.50)	(97.31, 568.04)	(57.98, 120.72)	(46.23, 64.52)	(37.08, 36.46)
		0.50	(54.58, 1140.51)	(19.31, 80.22)	(13.15, 21.67)	(11.26, 13.97)	(9.55, 9.03)
		0.75	(11.37, 161.58)	(5.33, 13.19)	(4.21, 5.48)	(3.82, 3.88)	(3.45, 2.90)
		1.00	(3.32, 18.60)	(2.23, 2.87)	(2.00, 1.74)	(1.89, 1.43)	(1.80, 1.19)
		1.50	(1.20, 2.31)	(1.11, 0.45)	(1.08, 0.32)	(1.06, 0.27)	(1.05, 0.23)
		2.00	(1.01, 0.15)	(1.00, 0.05)	(1.00, 0.02)	(1.00, 0.00)	(1.00, 0.00)
0.6400	2.5	0.25	(391.63, 19179.75)	(102.49, 762.14)	(58.52, 131.67)	(45.64, 67.58)	(36.71, 35.84)
		0.50	(73.50, 1992.49)	(22.95, 140.25)	(14.30, 25.87)	(11.99, 15.64)	(10.09, 9.67)
		0.75	(18.81, 467.62)	(6.14, 18.74)	(4.61, 6.52)	(4.07, 4.34)	(3.62, 3.07)
		1.00	(4.57, 56.38)	(2.49, 5.62)	(2.08, 2.02)	(1.95, 1.55)	(1.85, 1.25)
		1.50	(1.26, 3.50)	(1.11, 0.56)	(1.07, 0.30)	(1.04, 0.23)	(1.02, 0.13)
		2.00	(1.02, 0.25)	(1.00, 0.06)	(1.00, 0.01)	(1.00, 0.00)	(1.00, 0.00)
0.4444	3.0	0.25	(248.25, 3859.90)	(109.01, 1147.43)	(59.25, 158.25)	(45.97, 71.24)	(36.15, 35.48)
		0.50	(171.01, 26861.18)	(26.72, 263.80)	(15.71, 36.41)	(12.66, 17.04)	(10.52, 9.99)
		0.75	(89.20, 17373.17)	(7.72, 74.08)	(5.03, 7.88)	(4.33, 4.87)	(3.83, 3.27)
		1.00	(10.97, 474.13)	(2.83, 18.56)	(2.20, 2.61)	(2.02, 1.71)	(1.88, 1.28)
		1.50	(1.45, 17.18)	(1.11, 0.67)	(1.05, 0.30)	(1.02, 0.18)	(1.00, 0.00)
		2.00	(1.03, 1.29)	(1.00, 0.08)	(1.00, 0.01)	(1.00, 0.00)	(1.00, 0.00)

Conclusions

In this paper, we investigate the ARL and SDRL performances of the DS $\bar{X}$ chart with known and estimated parameters when the underlying distributions are Weibull, lognormal and gamma. The results show that the DS $\bar{X}$ chart with known and estimated parameters is seriously influenced by skewed distributions, especially for small number of Phase-I samples and sample sizes. At least 80 Phase-I samples are needed for the DS $\bar{X}$ chart with estimated parameters to behave similarly as the known-parameter chart; however, its performances are still unfavourable for moderate and large γ. Also, the in-control and out-of-control ARL and SDRL values are absolutely undesirable as the level of skewness γ increases. This suggests that the chart's parameters (n_1, n_2, L_1, L, L_2) specially designed for the DS $\bar{X}$ chart with known and estimated parameters under normal distribution, are inappropriate to be used in the skewed populations. Therefore, when the underlying distributions are skewed, new theories and chart's parameters for the DS $\bar{X}$ chart with known and estimated parameters need to be developed in future research.

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