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TIME DOMAIN MODELLING OF AN ELECTROMAGNETIC PULSE INTERACTED WITH A LOSSY DIELECTRIC HALF SPACE

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In this work, the electromagnetic interaction of an electromagnetic pulse with a lossy dielectric half space is studied theoretically and numerically. EMP is analysed in both frequency and time domains during its penetration through half space. Mathemathical theory of the interaction is realized by using frequency domain considerations and is based on Cartesian frame system. The dispersive effect of the half space on EMP is calculated via a matrix model. Time domain results are carried out using inverse Fourier Transform of frequency domain computations. A numerical inversion technique is used for attaining high accuracy on time domain results. A computer code is generated to calculate the electric and magnetic fields at any point of the half space for different elevation angles.

Keywords: Electromagnetic Pulse, Inverse Fourier Transform, Half Space.

Introduction

The interaction (reflection and transmission) of electromagnetic waves with half spaces and multilayered structures are of interest in many technological applications like communication engineering, soil sciences, electromagnetic compatibility and biomedical engineering. The problem of interaction of electromagnetic waves with multilayered dielectrics, stratified medium and half spaces have been discussed for normal incidence, different number of layers and vertical or horizontal polarizations[1-7]. A considerable amount of researches have been realized in frequency domain but limited studies have been carried out evaluating the time domain response. In frequency domain analysis, reflection and transmission coefficients [1-6] or some different matrix models[7-8] are taken into account for calculating transmission and reflection of electromagnetic fields. Time domain results are obtained by a numerical inverse Fourier Transform of frequency domain response. Barnes and Tesche[2] computed the electromagnetic field reflection from a lossy half space in time domain by using a close form of a frequency domain reflection coefficient. Poljak and Doric [3] studied on the transient behaviour of a straight line placed in a dielectric half space effected by EMP. The formulation is based on Transmission line approach, Fresnel reflection and transmission coefficients are used to analyse the effects of the half space. Perry and Rothwell [4] evaluated the transient electromagnetic wave reflection from layered dispersive materials and a method of converging series of reflection coefficients in time domain is used for analysis. Zeng and Delisle analyzed time domain analysis of the wave reflection from stratified medium [5] and by a numerical inversion technique based on inversion of Laplace Transform. Thakur and Holmes [6] introduced an easy way to determine the reflection and transmission coefficients of a plane electromagnetic wave interacted with a multilayered dielectrics via a matrix model. Another matrix model is used by Ari and Blumer [7] to study the transmission of NEMP through stratified medium and determine the shielding effectiveness of layered structure for different elevation angles and fundamental quantities (σ_r, σ_i) of half space. Khamy and Gendy[8] investigated the penetration of EMP in lossy dielectric media for various values of conductivity.

In this study, interaction of an EMP with a lossy dispersive dielectric half space is considered. Firstly, mathemathical theory of the interaction is realized in frequency domain. Then, the frequency domain results are converted to time domain results via inverse Fourier Transform [9-11]. To obtain the inverse Fourier Transform efficiently, some numerical inversion tecnique has to be applied to realize the inverse transform.

A computer code is generated for calculations and simulations. Electric and magnetic fields are analyzed and determined. Simulations are performed for different elevation angles and depths.

Theoretical Model of Interaction

The physical model of considered system in Cartesian frame is shown in Figure 1. The incident and transmitted electromagnetic fields and angular parameters of interaction are also shown in figure 1. The direction of incident EMP is defined by elevation(φ) and azimuth(Φ) angles. The polarization angle(θ) gives the deviation of the EMP from y-z plane.

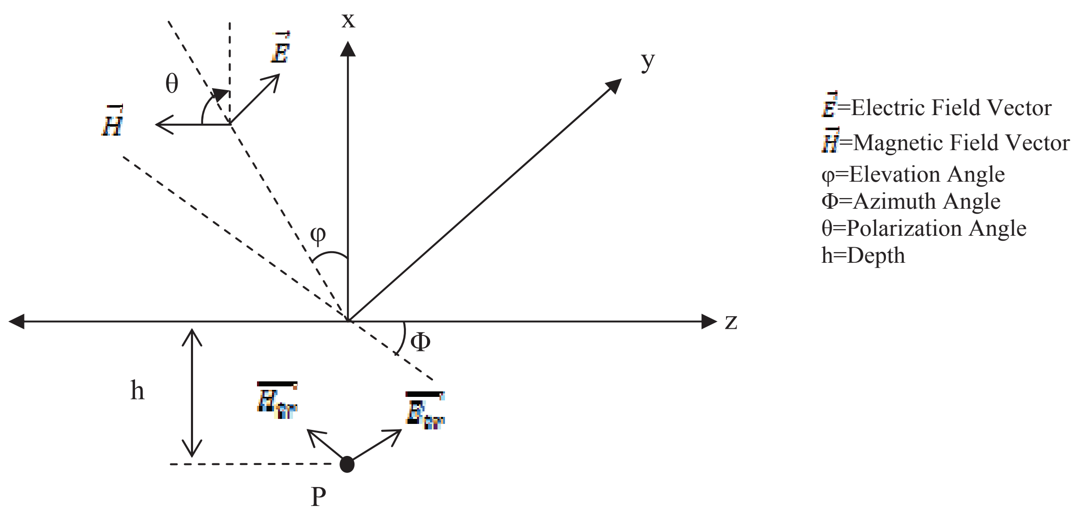


Figure 1. Interaction system.

The half space matrix is defined to treat the wave propagation in the half spaces. A halfspace matrice is illustrated for each half space(upper and lower). Electric and magnetic field vectors on z-axis direction are chosen as plotting components at the boundary of halfspaces. These two half matrices is used to determine the interaction of electromagnetic wave with lossy dielectric half space by generating the linear connection of tangential electromagnetic field vectors and defined plotting components. Half space matrices(upper($[M_U]$) and lower($[M_L]$)) are given below;

$$[M_U] = \begin{bmatrix} 1 & 0 \\ 0 & \frac{Zk}{k_x} \\ 0 & 1 \\ \frac{-k}{Zk_x} & 0 \end{bmatrix} \qquad [M_L] = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{Zk}{k_x} \\ 0 & 1 \\ \frac{k}{Zk_x} & 0 \end{bmatrix} \tag{1}$$

According to Eq.2 half matrices are defined by wave impedance and wave number of the half spaces and complex frequency ($s = \Omega + jw$). Wave impedance and wave number, depend on the fundamental quantities conductivity(σ_i), permittivity(ε_i), permeability(μ_i), are given below;

$$Z_i = \sqrt{\frac{s\mu_i}{\sigma_i + s\varepsilon_i}} \tag{2}$$

$$k_i = \sqrt{[-s\mu_i(\sigma_i + s\varepsilon_i)]} \tag{3}$$

The incident plotting components in the half space are determined below;

$$E_z = E(s)D_E(\theta, \varphi, \Phi)e^{-jkr} \tag{4}$$

$$H_z = H(s)D_H(\theta, \varphi, \Phi)e^{-jkr} \tag{5}$$

where D_E and D_H are the angle functions and $\vec{r} = (\vec{x}, \vec{y}, \vec{z})$ and it is defined by the distance of observation point to the boundary of halfspaces and calculated as;

$$r = h_i = x_p - x_i \tag{6}$$

the other tangential components of incident wave are determined by using lower half space matrice;

$$\begin{bmatrix} E_t \\ H_t \end{bmatrix}_{mo} = \begin{bmatrix} E_z \\ E_y \\ H_z \\ H_y \end{bmatrix}_{mo} = [M_L] \begin{bmatrix} E_z \\ H_z \end{bmatrix}_{mo} \tag{7}$$

Electric and magnetic fields at the observation point are calculated by the linear equation, is used the plotting components at the interface, is shown below;

$$\begin{bmatrix} E_t \\ H_t \end{bmatrix}_{mo} = -[M_U] \begin{bmatrix} E_z \\ H_z \end{bmatrix}_{so} + [M_L] \begin{bmatrix} E_z \\ H_z \end{bmatrix}_{hz} \tag{8}$$

Theory of Inverse Laplace Transform

The Laplace Transform of a real function f(t) ($t \geq 0$) is given by

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt \tag{9}$$

F(s) and f(t) are defined as the the image function in the complex frequency domain and the real valued function in time domain of each other. Inverse time dependent function f(t) is given by the inversion formula:

$$f(t) = \frac{1}{2\pi i}\int_{\Omega-j\infty}^{\Omega+j\infty} F(s)e^{st}ds \tag{10}$$

The real and imaginary parts of F(s) is alternatively shown as

$$f(t) = \frac{e^{\Omega t}}{\pi}\int_0^{\infty} [Re\{F(s)\}coswt - Im\{F(s)\}sinwt]\,dw \tag{11}$$

where $s = \Omega + jw$ represents the Laplace variable. Then , equation (7) and (9) is replaced by the cosine or sine transform pair[11] .

$$f(t) = \frac{2e^{\Omega t}}{\pi}\int_0^{\infty} Re\{F(s)\}coswtdw \tag{12}$$

$$f(t) = \frac{-2e^{\Omega t}}{\pi}\int_0^{\infty} Im\{F(s)\}sinwtdw \tag{13}$$

Electric and magnetic fields may be understood as causal time functions (*f(t)=0 for t<0*). Causal time functions can be represented by the real expression where the transient response is evaluated by using the imaginary part of frequency domain solution only. Fourier integral is evaluated on the real axis for $\Omega = 0$. F(w), is the fourier transform of f(t), is the computation result determined from the interaction of an electromagnetic wave with half space. For $\Omega = 0$ equation (13) is revised below:

$$f(t) = -\frac{2}{\pi}\int_0^{\infty} Im(F(w))sin(wt)dw \tag{14}$$

Simulations

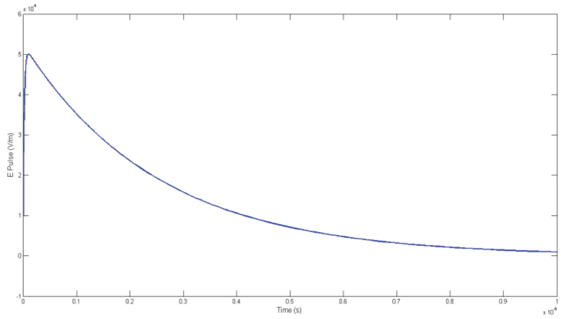
EMP-wave is modelled by a plane electromagnetic wave and described by a double exponential waveform:

$$E(t) = E_0\left(e^{-\alpha t} - e^{-\beta t}\right) \tag{15}$$

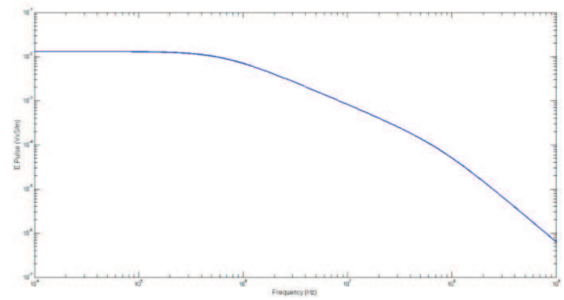
Mathemathical theory of the system is realized in frequency domain. So, waveform equation is transferred into frequency domain by using Laplace transform

$$E(s) = E_0\left(\frac{1}{s+\alpha} - \frac{1}{s+\beta}\right) \tag{16}$$

where $s = jw$ ($\Omega = 0$) denotes. The pulse parameters are $\alpha = 4x10^6 s^{-1}$, $\beta = 4.76x10^8 s^{-1}$ and $E_0 = 5x10^4\ V/m$. The EMP in time and frequency domain is shown in Figure 1.



a)



b)

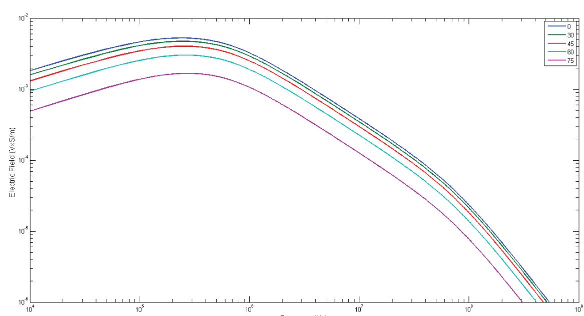
Figure 2. Double exponential incident Electric Field in a) time domain, b) Frequency domain..

EMP is assumed to interact with the half space that has a conductivity of $\sigma = 0.0001 mho/m$ and a dielectric constant $\varepsilon_r = 1$ with an incidence angle $\varphi = 0$ and with a polarization angle $\theta = 0.$

Results

Electric and magnetic field components are observed for different elevation angles. A matrix model is used to calculate the tangential components of electric and magnetic fields in the half space and electric and magnetic fields are both plotted.

Transmitted EMP is evaluated for different elevation angles ($\varphi = 0, 15, 30, 45, 60, 75$) with $\sigma = 0.0001 mho/m$, $\varepsilon_r = 10$ and h=5 m in frequency domain (Figure 3) and time domain(Figure 4). Due to the elevation of the incident pulse, a vertical magnetic field component is appeared instead of the horizontal magnetic field component. Because of the close results for elevation angles $\varphi = 0$ and $\varphi = 15$ in frequency domain, the results for $\varphi = 15$ is hidden in Figure 3a and 3b.



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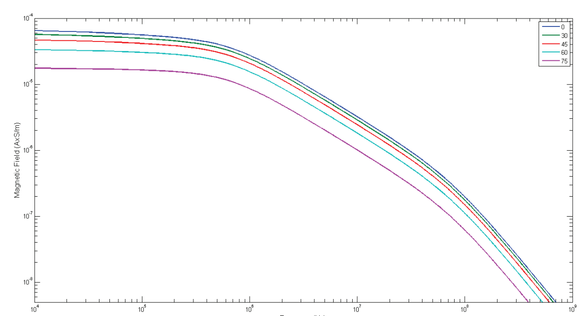

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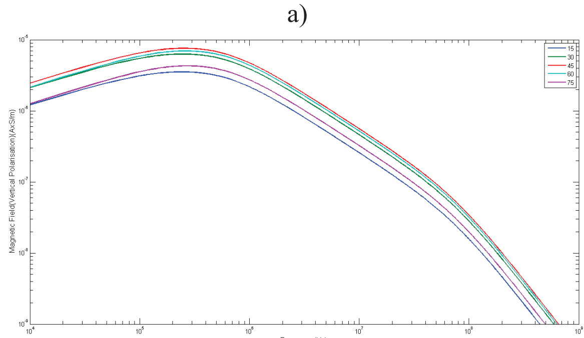

c)

Figure 3. Transmitted EMP for various elevation angles in frequency domain
a) Electrical Field b) Magnetic Field (Horizontal Polarization) c) Magnetic Field (Vertical Polarization).

In frequency domain, electric field results decreases with increasing of the elevation angle (Figure 3a). The decrement is much more at lower frequencies. For horizontal polarization of magnetic field, it is clear that the increasing elevation angle causes decreasing of horizontally polarized magnetic field results (Figure 3b). Also, it is pointed out that horizontal magnetic field outputs are greater than the incident magnetic field at elevation angles lower than $\varphi = 45$. For vertical polarization, maximum magnetic field result is calculated at $\varphi = 45$ in Figure 3c and magnetic field results continue increasing until $\varphi = 45$ and then the results are decreased with increasing elevation angle.

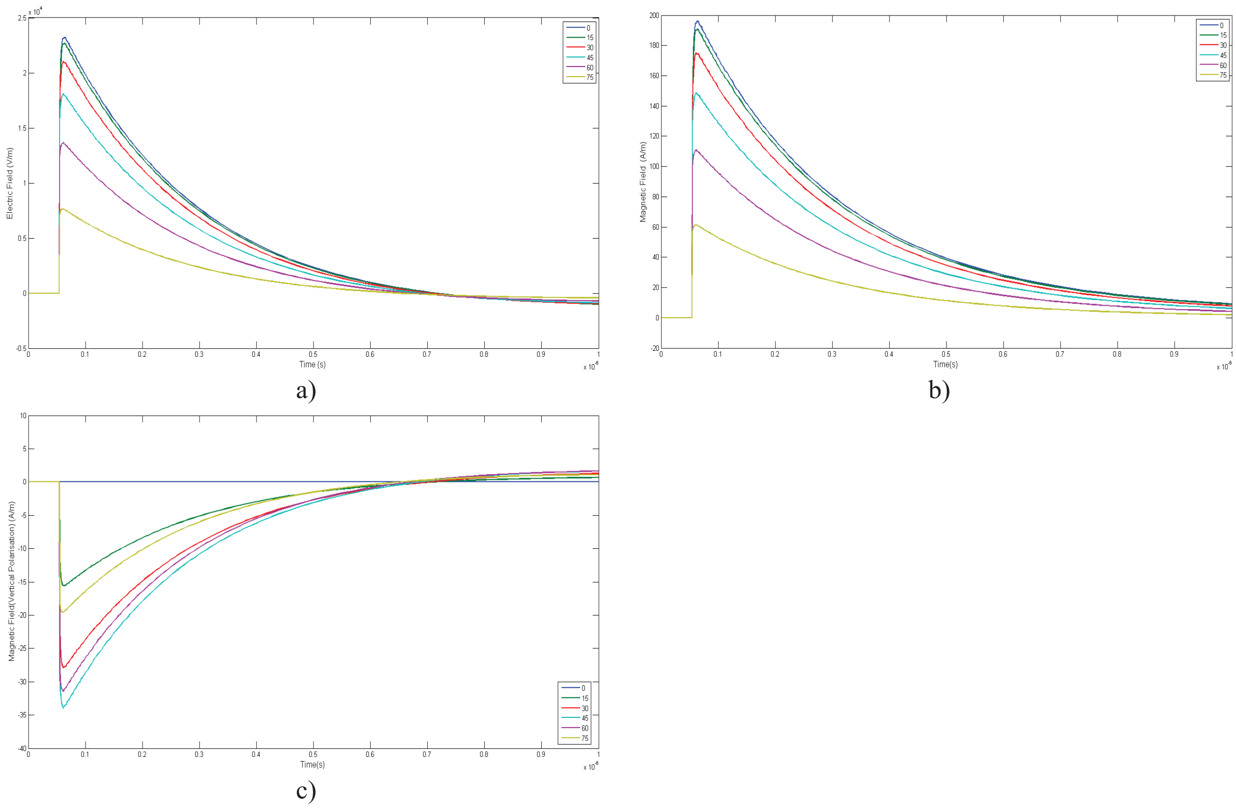

Figure 4. Transmitted EMP for various elevation angles in time domain a)Electrical Field b) Magnetic Field (Horizontal Polarization) c)Magnetic Field (Vertical Polarization).

In time domain, electric field results (Figure 4a) are decreasing with increasing elevation angles. For $t \approx 54ns$ the transmitted EMP appears at point P and from $t \approx 6.6\mu s$ for $\varphi = 75$ and $t \approx 7.25\mu s$ for $\varphi = 0$ electric field results turn to negative. For horizontal polarization, magnetic field results decrease with increasing elevation angles(Figure 4b). And for vertical polarization, the transmitted pulse values are negative due to the polarization of incident pulse for all elevation angles. Maximum magnetic field output is determined at $\varphi = 45$ and magnetic field results continue increasing until $\varphi = 45$ and then the results are decreased with increasing elevation angle(Figure 4c).

Conclusions

A theoretical and numerical study of an EMP penerated through a lossy half space is performed for both in frequency and time domain. Electric and magnetic field components are calculated by using half space matrices and a numerical inversion tecnique is applied on the frequency domain results to provide efficient results in time domain.

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