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MAPPING AND EVALUATING PEDAGOGICAL OPPORTUNITIES PROVIDED BY SIMREAL+: A CASE STUDY IN MATHEMATICS EDUCATION

Said Hadjerrouit

University of Agder, Norway

SimReal+ is a digital tool that provides both functional and pedagogical opportunities to teach a wide range of mathematical topics. Functional opportunities in terms of drawing graphs, doing calculations, constructing figures, or executing mathematical algorithms quickly and correctly ensure a trouble-free interaction with the digital tool while pedagogical opportunities aim at supporting the learning process. However, an inappropriate use of the tool can obstruct the pedagogical opportunities that can be provided at different levels. The paper focuses on mapping and evaluating pedagogical opportunities of SimReal+. It uses a pedagogical map to classify the opportunities at three different levels: The task level, the classroom level and interpersonal aspects, and the mathematical subject level.

Keywords: Affordance, MAS, Pedagogical opportunities**,** SimReal+.

Introduction

Digital tools in mathematics should provide both functional and pedagogical opportunities to enhance the learning and teaching process. While functional opportunities are self-evident requirements for any digital tool, pedagogical opportunities and their actualisation are less evident in educational settings (Hadjerrouit, 2015; Mayes & Fowler, 1999; Nokelainen, 2006; Tselios, Avouris, & Komis, 2008). In many cases, the impact of functional opportunities may be limited when it comes to pedagogical use of the tool in authentic educational settings. In other cases, pedagogical opportunities will only be visible under certain specific conditions, especially when an explicit pedagogy guides the use of the tool in classroom. Following this line of argumentation, Pierce and Stacey (2010) proposed a taxonomy of pedagogical opportunities provided by Mathematics Analysis Software (MAS), such as Computer Algebra Systems (CAS), graphical calculators, dynamic geometry, or SimReal+. The taxonomy is implemented in the form of a ‘pedagogical map’ that classifies the opportunities according to whether they arise from the mathematical task level, the classroom level and interpersonal aspects, or from the mathematical subject level. The taxonomy is used to actualize pedagogical opportunities provided by MAS in educational settings. The intention of this paper is to use the taxonomy to report on pedagogical opportunities of SimReal+ in a form that is beneficial for teachers and students.

SimReal+

SimReal+ is an interactive visualization tool for teaching and learning a wide range of mathematical subjects. The basic idea of SimReal+ is that visualizations are powerful mechanisms for learning mathematics and explaining difficult topics. According to Arcavi (2003), visualization is the ability to use pictures, graphs, animations, images, and diagrams on paper or with digital tools with the purpose of communicating information, thinking about and advancing understandings. There is a huge interest in visualization in mathematics education (McKenzie, & Clements, 2014; Presmeg, 2014). Textbooks are filled with pictures, diagrams, and graphs. However, there is little empirical work on visualizations using digital tools in educational settings (Macnab, Phillips, & Norris, 2012). Likewise, there is little research on SimReal+ and its use in mathematics education (Brekke, & Hogstad, 2010; Hadjerrouit, 2015). SimReal+ is a visualization tool that uses a graphic calculator, video lectures and streaming, video and interactive simulations to teach mathematics. It also provides exercises and applications in various areas of mathematics (Brekke, & Hogstad, 2011). Figure 1 shows the main components of SimReal+.

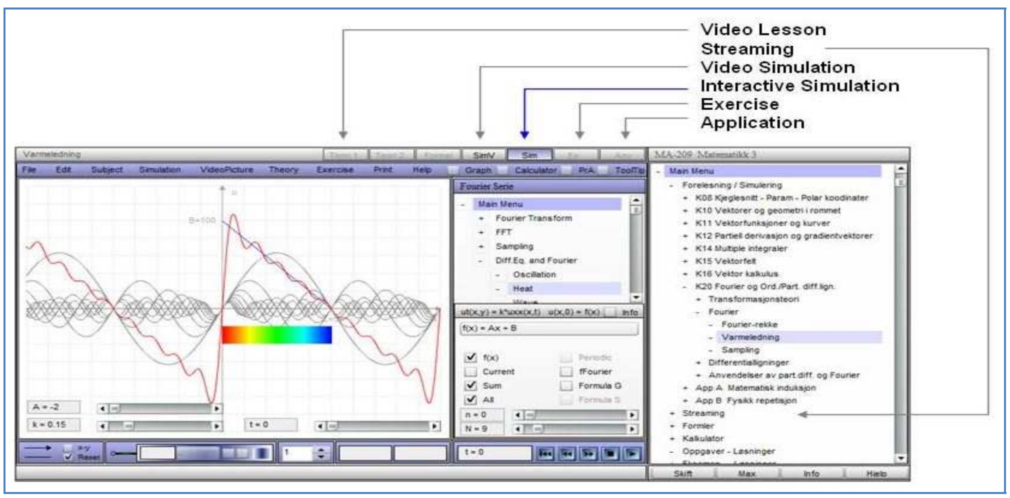

Figure 1. SimReal+ main components

Theoretical Framework

SimReal+ as a visualisation tool is a Mathematics Analysis Software (MAS) that supports the execution of arithmetic calculations, symbolic algebra manipulations, statistics calculations, visualization of data and functions, construction of geometric figures, or simulation of mathematical functions. There are two uses of SimReal+: functional and pedagogical use. The former refers to functional opportunities provided by the tool in terms of performing calculations, drawing graphs, constructing figures, or execute mathematical algorithms. The term opportunity is closely related to the concept of affordance, which was originally proposed by Gibson (1986). It refers to the relationship between an object's physical properties and the characteristics of a user. The concept was introduced to the Human-Computer-Interaction (HCI) community by Norman (1988), who defined an affordance as the design aspect of an object which suggests how the object should be used, a visual clue to its function and use. According to Pierce and Stacey (2010), functional opportunities provide a foundation for pedagogical opportunities for teachers to make changes to what mathematics is taught, to how mathematics is assessed, and as to how it is learned. The pedagogical opportunities provided by SimReal+ can be analyzed at three different levels (Figure 2):

- The task level, that is the tasks set for the students
- The classroom interaction level
- The subject level, that is the area of mathematics being taught

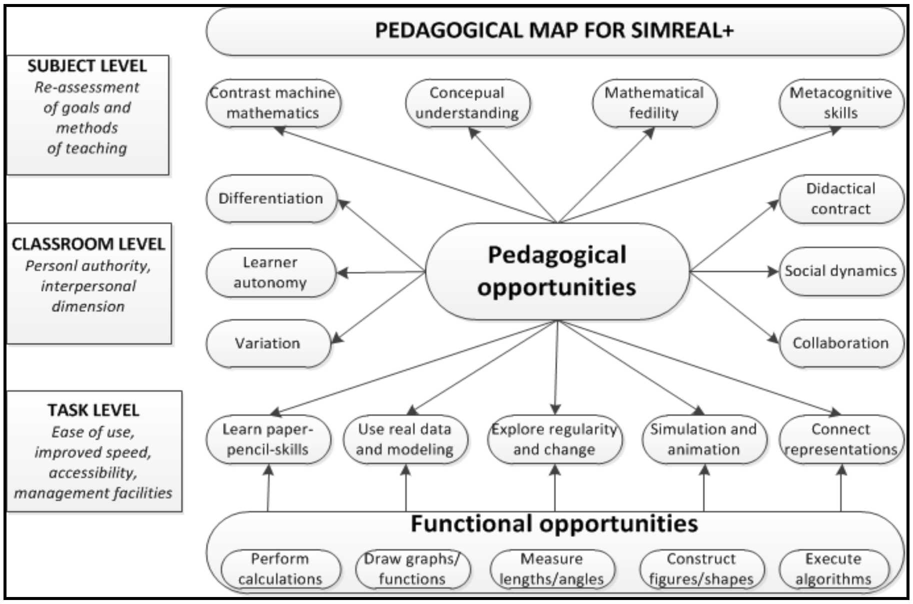

Figure 2. Pedagogical map for SimReal+

There are a number of pedagogical opportunities that can be provided at the task level, e.g., using the tool to freely build and transform mathematical expressions that support the learning of pen-and-paper skills, collecting real data and create a mathematical model, using a slider to vary a parameter or drag a corner of a triangle in geometry software, moving between symbolic, numerical, and graphical representations, simulating mathematical concepts, or exploring regularity and change. Opportunities at this level are supported by technological affordances in terms of ease-of-use, accessibility, improved speed, and management facilities.

Likewise, a number of pedagogical opportunities are provided at the classroom level (Stacey & Pierce, 2010). These result in changes of interpersonal dimensions, such as change of teachers' and students' role, less teacher-directed instruction and more student-oriented, computer as «new» authority in assessing student learning, resulting in students taking greater control over their own learning (learner autonomy), change of social dynamics, and more focus on collaborative learning and group work, as well as change of the didactical contract (Brousseau, 1998). Variation in teaching and differentiation are other opportunities offered by digital tools at this level (Hadjerrouit & Bronner, 2014).

Finally, three types of pedagogical opportunities can be provided at the mathematical subject level (Stacey & Pierce, 2010). The first one is fostering mathematical fidelity, looking at differences between machine mathematics and ideal mathematics, and promoting faithfulness of machine mathematics (Zbiek, Heid, & Blume, 2007). The second opportunity is amplifying and reorganizing the mathematical subject. The former is accepting the goals to achieve those goals better. Reorganizing the mathematical subject means changing the goals by replacing some things, adding others, and reordering others. For example, in calculus there might be less focus on skills and more on mathematical concepts. In geometry, there might be emphasis on more abstract geometry, and away from facts, more argumentation and conjecturing (Ibid). Finally, it may be useful to support tasks which encourage metacognition, e.g., starting with an overview or real-world application, and using MAS to generate results.

Research Questions and Methods

This work focuses on students' evaluations of pedagogical opportunities provided by SimReal+. It involved 22 teacher students taking the course on digital tools in mathematics education in 2015. The students had different knowledge background both in mathematics and digital literacy. None of the students had prior experience with SimReal+. The work used a survey questionnaire with open-ended questions to collect data. Teaching activities over a period of two weeks focused on basic, elementary and advanced mathematics. Basic mathematics included games, such as dices, tower of Hanoi, and similar tasks. Elementary mathematics consisted of arithmetic and algebraic exercises. Advanced mathematics included measurement exercises, trigonometry, differentiation, and Fourier. The teaching activities included video lectures, simulations of basic, elementary, and advanced mathematics using SimReal+, and online teaching material. To evaluate students' perceptions of pedagogical opportunities provided by SimReal+, a survey questionnaire based on a five-point Likert scale from 1 to 5 was used, where 1 was coded as the highest and 5 as the lowest (1="Strongly Agree"; 2 = "Agree"; 3 = "Neither Agree or Disagree"; 4 = "Disagree"; 5= "Strongly Disagree"). The average result (Mean) and Standard Deviation were calculated. The survey included 73 statements covering functional and pedagogical opportunities. The students were asked to comment each of the statements in their own words. In addition, they were required to address 5 open-ended questions to express in their own words what they think on specific issues of SimReal+. The responses to open-ended questions were analyzed qualitatively. Given this background, this work aimed at answering the following questions:

- What are the students' perceptions of pedagogical opportunities provided by SimReal+ at the task, classroom, and subject level?
- What are the pedagogical implications at the task, classroom, and subject level?

Results

Pedagogical Opportunities at the Task Level

At the task level, students indicated that SimReal+ visualizations and simulations provide opportunity to help them gain knowledge that is otherwise difficult to acquire. Likewise, the issue of using real data and practical applications obtained an average result in terms of design quality, and that the advanced exercises were not difficult to understand. Some students think that many tasks lack explanation, but once one understands what should be done, the exercises show a high degree of quality that promotes knowledge acquisition.

Furthermore, most students believed that SimReal+ is very useful and essential when it links video lessons, simulations, live streaming of lessons, and exercises. It can help students to understand and see connections between different areas of mathematics. Some students also believed that SimReal+ is congruent with paper-pencil techniques when solving mathematical tasks, but it does not help much to update and renew mathematical knowledge.

Finally, SimReal+ is not considered as a better tool to learn mathematics than GeoGebra, which has more pedagogical advantages, such as better individual differentiation and that it takes more time for a number of students to learn the mathematical topic using SimReal+ than a textbook. Nevertheless, SimReal+ is considered as a good supplement to lectures.

Pedagogical Opportunities at the Classroom Level

At the classroom level, most students agreed that SimReal+ is fully appropriate to use as an alternative to achieve variation in teaching mathematics, amplify the learning outcome and comprehension of notions, for example graphs of functions and derivatives. Most students think that this is the best component of SimReal+, because it offers a lot of opportunities to work with visualizations and animations. Likewise, most students think that the tool provides more autonomy to do mathematics. Even though SimReal+ provides little mathematical background of the simulations, the possibility to deal with the simulations offers the opportunity to automatically develop the understanding of notions.

Students also revealed that simulations of mathematical concepts are highly motivating and present problems in a much exciting way that verbal descriptions would do. In addition, the students were satisfied with SimReal+ in terms of individualization and differentiation, and choice of level of difficulty, while enabling them to work at their own pace, which is a motivational factor in keeping students engaged in mathematics. However, there is not a variety of scaled-difficulty exercises, e.g., there are no equations that would need sophisticated algebraic manipulations, but only very simple ones. Most students also still think that SimReal+ does not fully allow working independently from teacher assistance. Also textbooks are still needed when using SimReal+ in classroom.

Finally, the vast majority of the students did not find deliberate collaborative tools installed. Nevertheless, it is worth noting that some simulations stimulate communication, e.g. how somebody else perceives the visual illustrations and work with them.

Pedagogical Opportunities at the Subject Level

At the subject level, most students agreed that SimReal+ has a high degree of mathematical content in terms of mathematical correctness and representation of mathematical properties and operations, e.g., formulas, functions, graphs, and geometrical figures. Basically, SimReal+ presents the mathematics in a principally correct way, but it can be done more, for example to solve equations of second grad, or logical games, etc. Most students also believed that visualizations are easy to understand. Although SimReal+ seems to have a wide range of mathematical topics, it is rather hard for some students to figure out how one can deal with them, for example, how to manipulate parameters, or how one can see the movement that takes place, because the screen has only a limited space.

Discussion

Given the students' experiences and evaluations, some implications are drawn from the results for the three levels that emerged from the pedagogical map. At the task level, the strong side of SimReal+ is that it allows a multiple representations of mathematical tasks using video lessons, simulations, and visualizations. This is clearly an advantage to acquire mathematical skills and explore variation and regularity. Another strong side of SimReal+ is that the provided exercises are well-designed and contain different levels of difficulty, and that the tool is sound to the underlying mathematical properties. One weak side of Simreal+ is that it does not give feedback, and it does not provide several assessment modes. In addition, the user interface of SimReal+ is still not intuitive and straightforward. Simreal+ has been recently improved to satisfy students' needs by integrating more appropriate exercises with different of difficulty. This a good direction, but research still remains to be done to make SimReal+ more appropriate to its objective.

At the classroom level, SimReal+ allows variation and differentiation in teaching, but it does not provide opportunities to customize the tool. Furthermore, SimReal+ does not promote group work, perhaps because collaboration tools are not integral parts of the tool. One way for the teacher to stimulate group work is to design collaborative and group tasks. Likewise, SimReal+ does not allow a high degree of student autonomy, perhaps due to the limitation of the feedback provided. Hence, teacher assistance is still needed to make learning happen, and the didactical contact (Brousseau, 1998) does not need to be radically changed when using SimReal+. As a result, a number of improvements need to be made in order for SimReal+ to foster collaborative learning and provide useful feedback to the students.

At the subject level, SimReal+ provides pedagogical opportunities to faithfully represent mathematical expressions and formulas. However, SimReal+ does not automatically provide support for the teacher to adjust goals, spend less time on routine skills, and more time on concepts. This is true for weak students, struggling with understanding mathematical concepts. Teacher help is still needed, since students showing procedural skills do not necessarily have a conceptual understanding of mathematics. On the other hand, SimReal+ can be used to produce "unexpected" mathematical results as catalyst for rich mathematical discussion. Hence, teachers can deliberately use the constraints and limitations of SimReal+ to foster students' mathematical thinking (Zbiek, Heid, & Blume, 2007).

Conclusion

Despite the limitations of the study, it has been possible to make some reasonable interpretations of the results based on the theoretical framework and draw some recommendations for using SimReal+ in mathematics education. Firstly, the results show that SimReal+ provides both functional and pedagogical opportunities for teaching and learning mathematics. Nevertheless, research remains to be done to make SimReal+ pedagogically fully usable in authentic educational settings. In addition, learning to use a new digital tool like SimReal+ may be demanding in terms of efforts and time, making the process of instrumental genesis more complex (Trouche, 2004).

References

1. Arcavi, A. (2003). The Role of Visual Representations in the Learning of Mathematics. *Educational Studies in Mathematics*, *52*(3), pp. 215-241.
2. Burden, K., & Atkinson, S. (2008). Evaluating Pedagogical Affordances of Media Sharing Web 2.0 Technologies: A Case Study. *Proceedings of ascilite* 2008, pp. 121-125. Melbourne, Australia.
3. Bokhove, K., & Drijvers, P. (2010). Digital Tools for Algebra Education: Criteria and Evaluation. *International Journal of Mathematics Learning*, *15*, pp. 45-62.
4. Brekke, M., & Hogstad, P.H. (2010). New Teaching Methods – Using Computer Technology in Physics, Mathematics, and Computer Science. *International Journal of Digital Society*, *1*(1), pp. 17-24.
5. Drijvers et al. (2010). Integrating Technology into Mathematics Education: Theoretical Perspectives. In C. Hoyles & J.-B. Lagrange (Eds.). *Mathematics and Technology-Rethinking the Terrain*. Berlin: Springer.
6. Gibson, J. (1986). *The Ecological Approach to Visual Perception.* New York: Lawrence Erlbaum Associates.
7. Hadjerrouit, S. (2015). Evaluating the Interactive Learning Tool SimReal+ for Visualizing and Simulating Mathematical concepts. *Proceedings of CELDA 2015,* pp. 101-108.
8. Hadjerrouit, S., & Bronner, A. (2014). An Instrument for Assessing the Educational Value of Aplusix for Learning School Algebra. In M. Searson & M. Ochoa (Eds.). *Proceedings of SITE 2014*, pp. 2241-2248. Chesapeake, VA: AACE.
9. McKenzie, K., & Clements, A. (2014). Fifty Years of Thinking about Visualization and Visualizing in Mathematics Education: A Historical Overview. In M.N. Fried, & T. Dreyfus (Eds.). *Mathematics & Mathematics Education: Searching for Common Ground, Advances in Mathematics Education*, pp. 177-192. Berlin: Springer.
10. Macnab, J.S., Phillips, L.M., & Norris, S.P. (2012). Visualizations and Visualization in Mathematics Education. In S.P. Norris (Ed.). *Reading for Evidence and Interpreting Visualizations in Mathematics and Science Education*, pp. 103–122. Rotterdam: Sense Publishers.
11. Nokelainen, P. (2006). An Empirical Assessment of Pedagogical Usability Criteria for Digital Learning Material with Elementary School Students. *Educational Technology & Society, 9*(2), pp. 178-197.
12. Norman, D. A. (2013). *Design of Everyday Things: Revised and Expanded.* New York: MIT Press.
13. Presmeg, N. (2014). Visualization and Learning in Mathematics Education. In S. Lerman (Ed.). *Encyclopedia of Mathematics Education*, pp. 636-640. Berlin: Springer.
14. SimReal+ Web site: http://grimstad.uia.no/perhh/phh/video/video.htm
15. Stacey, K., & Pierce, R. (2010). Mapping Pedagogical Opportunities Provided by Mathematics Analysis Software. *International Journal of Mathematics Learning*, *15*, pp. 1-20.
16. Trouche, L. (2004). Managing the Complexity of Human/Machine Interactions in Computerized Learning Environments: Guiding Students' Command Process through Instrumental Orchestrations. *International Journal of Computers for Mathematical Learning 9*, pp. 281-307.