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EVALUATING THE PEDAGOGICAL OPPORTUNITIES PROVIDED BY APLUSIX FOR LEARNING SCHOOL ALGEBRA: A CASE STUDY IN MATHEMATICS EDUCATION

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Aplusix is a digital tool that helps students to learn school algebra. It provides both technological and pedagogical opportunities. Technological opportunities in terms of speed of use, accessibility, improved display for mathematical notations, and congruence with paper-pencil-techniques ensure a trouble-free interaction with the mathematical software while pedagogical opportunities aim at supporting the teaching and learning of algebra. The goal is to minimize the students' cognitive load resulting from the interaction with the software in order to free more resources for the learning process itself. Basically, technological opportunities provide a foundation for pedagogical opportunities, but an inappropriate use of the software can obstruct the pedagogical opportunities to enhance the quality of teaching and learning. The paper focuses on students' evaluation of the pedagogical opportunities offered by Aplusix. It uses a 'pedagogical map', which further classifies the opportunities according to the task level, the classroom level, and the mathematical subject level.

Keywords: Aplusix, MAS, Pedagogical map, Pedagogical opportunity.

Introduction

Educational software in mathematics should provide both technological and pedagogical opportunities to enhance learning and teaching processes. While technological opportunities are self-evident requirements for any digital technology, pedagogical opportunities and their actualization are less evident to teachers (Mayes & Fowler, 1999; Nokelainen, 2006; Tselios, Avouris, & Komis, 2008). In many cases, even if the digital tool is convenient, practicable, and usable for students, the impact of technological opportunities may be limited when it comes to the pedagogical use of the tool in authentic educational settings. This is the case of software designed without pedagogical goals, for example Spreadsheets. In other cases, the pedagogical affordance will only be visible when an explicit pedagogy guides the use of the technology in classroom (Ibid). Basically, the features of educational technologies are often technically usable but pedagogically poor. Following this line of argumentation, Pierce and Stacey (2010) proposed a taxonomy of pedagogical opportunities provided by Mathematics Analysis Software (MAS), such as Computer Algebra Systems, dynamic geometry or statistical packages. The taxonomy is implemented in the form of a '*pedagogical map*', which further classifies the opportunities according to whether they arise from the mathematical task level, the classroom level and interpersonal aspects, or from the mathematical subject level. The map is potentially powerful to actualize pedagogical opportunities provided by MAS in educational settings. The intention of this paper is to use the pedagogical map as a theoretical framework to report on potential pedagogical benefits of teaching algebra using Aplusix in a form that is useful for students. The reminder of this paper is structured as follows. First, the theoretical framework is outlined. Second, the research goals, objectives, and methods are presented. This is followed by the presentation of Aplusix. Then, the results and discussion are described. Finally, some remarks conclude the article.

Theoretical Framework

Mathematics Analysis Software (MAS) includes two types of software. Firstly, generic software, such as Spreadsheets, and Internet. Secondly, subject-specific software, such as software for analyzing and exploring algebraic expressions, graphs of functions, diagrammatic/geometric representations, etc. Examples of the latter are GeoGebra, Cabri, Graph plotters, TI-Interactive, Computer Algebra Systems, and Aplusix. MAS can support the execution of algorithmic processes, e.g., arithmetic calculations, symbolic algebra manipulations, statistics calculations, data display, function graph, and construction of geometric figures.

There are two uses of MAS, a functional and a pedagogical use. Basically, the functional use provides functional opportunities, such as speed of use, accessibility, improved display of mathematics notations, etc. Functional opportunities provide a foundation for pedagogical opportunities to make changes to what mathematics is taught, to how it is assessed, and as to how it is learned. According to Pierce and Stacy (2010), the pedagogical map enables the analysis of pedagogical opportunities at three different levels (Figure 1):

- The task level, that is the tasks that teachers will set for their students (the bottom row)
- Their classroom interaction level (the middle row)
- The subject level, that is the area of mathematics being taught (the top row)

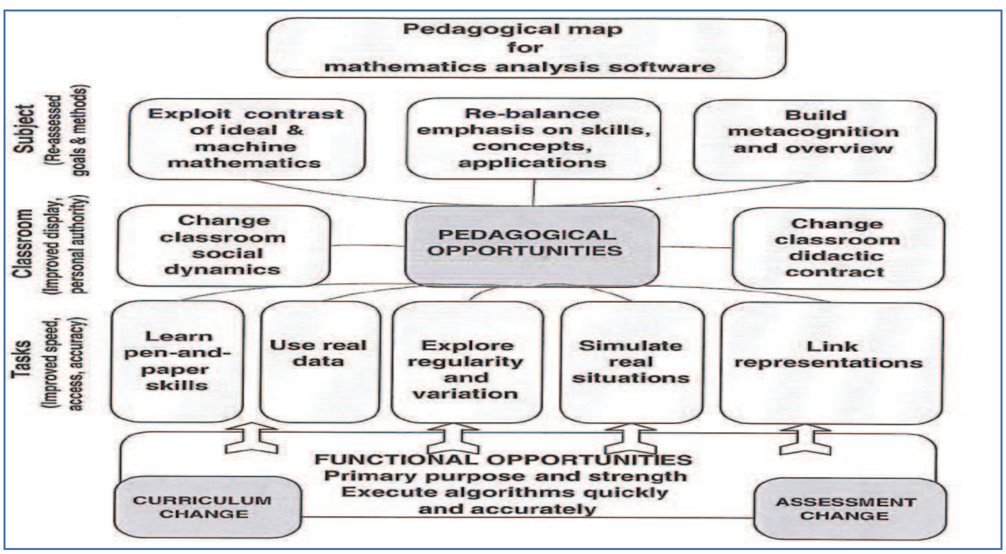

Figure 1. Pedagogical map for MAS at the task, classroom, and subject level (Pierce & Stacey, 2010, p. 6)

Research Questions and Methods

This work focuses on pedagogical opportunities provided by Aplusix. It explores the views of 15 students taking a course on digital tools in mathematics education. The course discusses theoretical background and various ways of using digital tools in mathematics education, impacts of digital tools on teaching and learning, and results from research and development projects. Aplusix was one of the digital tools being used in the course, in addition to GeoGebra and other pedagogical software. After a general presentation of the tool in classroom, the students used Aplusix to discover its potentialities, solve readily available exercises and problems, and create new problems and tasks. Aplusix was then available for use over a period 3-4 weeks until the final exam. The students used the four types of activities provided by Aplusix, that is training with feedback, test without feedback, self-correction mode after a test without feedback, and observation of previous work/activities. In addition, the students were asked to define their own exercises and problems using the Aplusix editor. Finally, the students were asked to evaluate Aplusix using the pedagogical map with slight modifications. The survey consisted of 15 questions, which are categorized in 3 groups of criteria to address the opportunities provided at the task, classroom, and subject level. The survey used a Likert Scale from 1 to 5, where 1=”Strongly Agree”, 2=”Agree”, 3=Neither Agree or Disagree”, 4=”Disagree”, and 5=”Strongly Disagree”. The survey provided an instrument for assessing Aplusix using a set of predefined criteria. Using the pedagogical map, this work aimed at answering the following questions:

- What are the students’ perceptions of pedagogical opportunities provided by Aplusix at the task, classroom, and subject level?
- To what extent are the pedagogical opportunities met and not met?
- What are the pedagogical implications at the task, classroom, and subject level that help to place Aplusix as an integral tool in teaching and learning algebra?

The Digital Tool Aplusix

Aplusix is a digital tool that assists students in learning school algebra (Chaachoua, Nicaud, Bronner, & Bouhineau, 2004). It allows students to freely build and transform algebraic expressions as they can do on paper. In addition, Aplusix provides management facilities and helps the teacher to add and modify the mathematical content. The tool has also readily available content. Aplusix is supposed to facilitate mathematical activities and enables students to express mathematical ideas. Furthermore, Aplusix provides several assessment modes (practice, test, observation), and that the tool gives direct feedback to the students’ actions (Figure 2). Aplusix applies to numerical calculations (integers, decimals, fractions, and square roots), expansion and simplification, factorization, as well as solving linear equations, quadratic equations, inequalities, and simultaneous equations. Aplusix contains four modes of activities for students:

- Training with feedback (practice), which means that an immediate feedback is given by the system showing whether two consecutive expressions are equivalent or not;
- Test without feedback;
- Self-correction mode after a test without feedback (delayed feedback after a test); and
- Observation of previous work/activities.

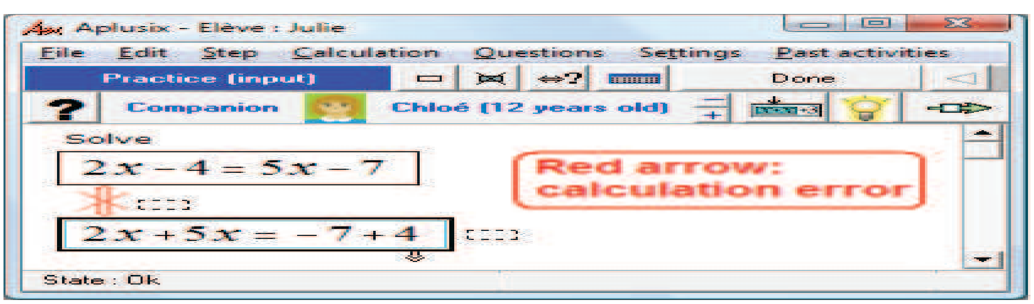

Figure 2. Aplusix showing a feedback “Red arrow: calculation error” (Aplusix Web site)

The feedback function of Aplusix is particularly important for solving equations, understanding the meaning of the equivalence sign, variables, and algebraic expressions (Maffei, Sabena, & Mariotti, 2009). As a result, Aplusix as MAS provides both technological and pedagogical opportunities and opportunities to help students learn school algebra.

Related Work

Several experiments with Aplusix have been carried out in different countries (Bouhineau et al, 2005; Nicaud et al, 2006; Nicaud, Bouhineau & Huget, 2004). These experiments included remediation piloted by researchers in Italy, remediation integrated into the regular functioning of classes in Brazil, collaborative learning in India, regular use during an entire year in France, and learning equations in Norway. On the basis of these experiments, researchers concluded that Aplusix has been shown to be a usable software tool, favoring the students' learning of school algebra (Bouhineau & al, 2005; Nicaud, Bouhineau, & Chaachoua, 2004). In addition, the cost of integrating Aplusix into the teaching of algebra is low. Furthermore, the students gained autonomy and improved their knowledge. Finally, Aplusix facilitated the teachers' work due to student autonomy and ready-made lists of exercises. Likewise, Hadjerrouit (2011) indicated that Aplusix shows potential for the learning of school algebra, even though not all students benefited equally well, and that Aplusix may have a positive impact on students' learning if conditions are met, such as students' prior knowledge in algebra, design of didactical situations, and integration of Aplusix in classroom. Finally, Hadjerrouit and Bronner (2014) used an evaluation instrument based on 5 groups of criteria, and a questionnaire with 15 questions to address these criteria. The results indicate that Aplusix shows potential for learning school algebra.

Results

Task Level

At the task level (Table 1), most students think that Aplusix gives direct feedback in the process of problem-solving (ITEM 5, MEAN=1.73, STD. DEV=0.594), and that the feedback is appropriate to its objectives and students' level, and can be adapted (ITEM 6, MEAN= 2.13, STD. DEV=0.990). Secondly, the overwhelming majority agreed that Aplusix enables the student to apply paper-and-pencil reasoning steps (ITEM 1, MEAN=2.00, STD. DEV=0.845), and that Aplusix facilitates students' algebraic activities (ITEM 2, MEAN=2.33, STD. DEV=0.617). Then, half of the students think that Aplusix enables to work on real algebraic problems involving calculations, which done by hand, are error-prone and time-consuming (ITEM 3, MEAN=2.47, STD. DEV=0.640). Likewise, Aplusix enables to explore regularity and variation in algebraic reasoning (ITEM 4, MEAN=2.47, STD. DEV=0.640). As a result, the pedagogical opportunities of Aplusix at this level are, firstly, the feedback it provides to students' actions, and secondly, Aplusix's congruence with paper-and-pencil techniques. Less actualized, but assessed as acceptable by the majority of the students, are pedagogical opportunities that facilitate algebraic reasoning, working on real problems, exploration of regularity, and multiple representations of algebraic content.

Table 1. Pedagogical opportunities at the task level

ITEM	MIN	MAX	MEAN	STD.DEV
1. Aplusix allows students to freely build and transform algebraic expressions as they can do on paper	1	4	2.00	0.845
2. Aplusix facilitates students' algebraic activities	1	3	2.33	0.617

3. Aplusix enables to work on real algebraic problems involving calculations, that, done by hand, are error-prone and time-consuming	2	4	2.47	0.640
4. Aplusix enables to explore regularity (generalizations) and variation in algebraic reasoning	1	3	2.47	0.640
5. Aplusix gives direct feedback to the students in the process of problem-solving	1	3	1.73	0.594
6. Aplusix's feedback is appropriate and can be adapted to the students' knowledge level	1	4	2.13	0.990

Classroom Level

At the classroom level (Table 2), students believed that the best pedagogical opportunity offered is the capability of Aplusix to provide opportunities for the teacher to make individual adjustments for each student, and define appropriate mathematical tasks (ITEM 9, MEAN=2.07, STD.DEV=0.458). Likewise, though to a lesser degree, the student is able to control his/her own learning of algebra (ITEM 8, MEAN=2.47, STD.DEV=0.915). Student autonomy and independence from teacher assistance are less actualized than originally expected, so that Aplusix alone, without the teacher, cannot exclusively become the "new authority" in assessing students' knowledge (ITEM 7, MEAN=2.60, STD.DEV=0.910). The traditional didactic contact between teacher and students is still important when using technology for teaching. Likewise, Aplusix does not stimulate much group work, discussion, cooperation, and sharing of knowledge with the class (ITEM 10, MEAN=2.93, STD.DEV=0.799).

Table 2. Pedagogical opportunities at the classroom level

ITEM	MIN	MAX	MEAN	STD.DEV
7. Aplusix enables a high degree of students' autonomy and independence from teacher assistance	1	4	2.60	0.910
8. Using Aplusix, the student is able to control his/her own learning of algebra	1	4	2.47	0.915
9. Aplusix provides opportunities for the teacher to make individual adjustments for each student, and define appropriate mathematical tasks	1	3	2.07	0.458
10. Aplusix stimulates students to cooperate, work in group, and share their learning with the class	2	4	2.93	0.799

Subject Level

The most obvious pedagogical opportunities at the subject level (Table 3) is the mathematical faithfulness and soundness of Aplusix to the underlying mathematical properties, e.g., conventional representations, sound operations (ITEM 14, MEAN =2.47, STD.DEV=0.640), and Aplusix capability to display algebraic expressions and formulas correctly (ITEM 15, MEAN=2.20, STD.DEV=0.862). Then, less than the majority believed that Aplusix provides opportunities for teachers to adjust goals, spend less time on routine skills, and more time on concepts (ITEM=12, MEAN=2.53, STD.DEV=0.743), followed by capability of the tool to enable teachers to use "unexpected" algebraic expressions as catalyst for rich mathematical discussion (ITEM 11, MEAN=2.67, STD.DEV=0.617). Less obvious is the opportunity that enables the teacher to link concepts through manipulation of algebraic expressions and use of multiple representations (ITEM 13, MEAN=2.73, STD.DEV=0.799).

Table 3. Pedagogical opportunities at the subject level

ITEM	MIN	MAX	MEAN	STD.DEV
11. Aplusix enable teachers to use "unexpected" algebraic expressions as catalyst for rich mathematical discussion	1	3	2.67	0.617
12. Aplusix provides opportunities for teachers to adjust goals, spend less time on routine skills, and more time on algebraic concepts and reasoning	2	4	2.53	0.743
13. Aplusix enables teachers to link concepts through manipulation of algebraic expressions and use of multiple representations	1	4	2.73	0.799
14. Aplusix is mathematically sound to the underlying algebraic properties (e.g. conventional representations, sound operations)	2	4	2.47	0.640
15. Aplusix is able to display algebraic expressions and equations correctly	1	4	2.20	0.862

Discussion

The pedagogical map used in this work draws on the work done by Pierce and Stacey (2010). It includes technological opportunities at three different levels (speed, access, accuracy, display, authority, subject, goals and methods) and pedagogical opportunities (Paper-pencil, multiple representation, real data, variation, simulation; classroom dynamics, didactic contract, skills, exploitation of ideal machine, metacognition), and their connectedness. The map provided insight into the use of Aplusix. It has proved to be useful to evaluate the pedagogical opportunities of Aplusix to learn school algebra at the task, classroom, and subject level.

The study investigated the extent to which the criteria are met. Using a survey questionnaire based on the issues of the pedagogical map, the students were asked to evaluate uses of Aplusix at three different levels. Given the students' answers, implications are drawn from the findings for the three levels that emerged from the map. This section focuses primarily on these levels, but will also review some of the common practical problems and issues identified.

Pedagogical Implications at the Task Level

The strong side of Aplusix is that it allows students to use the tool as they can do with paper-and-pencil techniques. This is clearly an advantage to acquire algebraic skills and solving equations step by step. Another strong side of Aplusix is that the provided exercises are well-designed and contain different levels of difficulty and that the tool is algebraically sound to the underlying mathematical properties. Another strong pedagogical opportunity provided by Aplusix is that it satisfies one of the key characteristics of modern intelligent tutoring systems, by giving instant feedback, and showing two black parallel bars, which mean that the current step is equivalent to the previous step, and two red parallel bars with an X mean that the current step is not equivalent to the previous step. Aplusix also provides several assessment modes (practice, test, observation, and review) in terms of formative assessment. It is also possible for the teacher to create motivating problems by using real data. Another advantage for the teacher is the use of data that are automatically generated by Aplusix. This helps to assess students' performance and evaluate students' problem-solving strategies.

As a result, Aplusix provided a number of pedagogical opportunities that help students to learn algebra. On the other hand, multiple representations of information are missing, making it difficult to explore variation and regularity. In addition, the feedback provided is not informative enough to allow conceptual, procedural, and corrective feedback. It could be improved by providing more information about the problem-solving process. Moreover, the feedback function does not provide appropriate feedback that is adapted to the students' level, in form of text or motivating animations or sound effects than the double red arrow indicating a wrong step. Aplusix has been recently improved by integrating a companion at three different levels to help through the problem-solving process. This a good direction, but research remains to be done to make the feedback more appropriate to its objective. Another way to improve the feedback is to allow graphical representations of algebraic expressions (Trgalova, & Chaachoua, 2009).

Pedagogical Implications at the Classroom Level

At the classroom level, Aplusix provides some pedagogical opportunities, e.g., making individual adjustments for each student, and defining mathematical tasks. However, Aplusix does not promote group work, discussion, and social dynamics, perhaps because communication and collaboration tools are not integrated into Aplusix. One way for the teacher to stimulate group work is to design collaborative and group tasks and exercises. Likewise, Aplusix does not allow students a high degree of student autonomy, perhaps due to the limitation of the feedback provided. Hence, teacher assistance is still needed to make learning happen. The didactical contact (Brousseau, 1998) does not need to be radically changed when using Aplusix in classroom. Nevertheless, some improvements need to be made to allow Aplusix become a "new authority" to a certain degree in evaluating the learning of algebra.

Pedagogical Implications at the Subject Level

At the subject level, Aplusix provides good pedagogical opportunities to faithfully represent algebraic expressions and formulas. However, Aplusix does not automatically provide support for the teacher to adjust goals, spend less time on routine skills, and more time on concepts. This is true for weak students, struggling with understanding algebraic concepts, such as equivalence of algebraic expressions, and relational understanding of the equal sign, in contrast to the operational understanding. Teacher help is still needed, since students do not necessarily have a relational understanding of the equal sign. On the other hand, Aplusix can be used to produce "unexpected" algebraic expressions as a catalyst for rich mathematical discussion, e.g., division by zero. Hence, teachers can deliberately use the constraints, limitations, and anomalies of Aplusix to foster students' algebraic thinking (Zbiek, Heid, & Blume, 2007). One limitation of Aplusix is that it does not take into account national differences in representing algebraic expressions. This may provoke critical reflections and thinking about local and national variations in mathematics practices.

Conclusions

The number of participants (*N=15*) is not sufficient to adequately support the generalization of the results. Hence, new cycles of experimentations and evaluations of Aplusix, are warranted to generalize the findings of the present work to ensure more validity and reliability. However, despite the limitations of the study, it has been possible to make some reasonable interpretations of the results and draw some recommendations for using Aplusix in mathematics education. Firstly, the results show that Aplusix provides both functional and pedagogical opportunities for teaching and learning algebra, and research experiments done in different countries confirm the potentialities of Aplusix. Still, research remains to be done to make Aplusix pedagogically fully usable in authentic educational settings. Secondly, learning to use a new technology may be demanding in terms of efforts and time, making the process of instrumental genesis more complex (Trouche, 2004). Hence, students and teachers need to familiarize themselves with the technology being used in classroom. Finally, Aplusix can be used in various ways depending on teacher's values, experiences, and perceptions of students' needs.

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