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MATHEMATICS, ART, AND COGNITIVE SCIENCES: SOME EXAMPLES OF CONNECTION

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Mathematics and Music, in the frame of art and science, have analogies and reciprocal connections, already known starting from the time of Pythagoras. Mathematics can also connect different artistic disciplines, such as music and visual arts. Recent developments in the field involve the mathematical modeling of orchestral performances and composition, as well as the study of effective strategies to translate music into images, and vice versa. The more meaningful examples of such a technique can be explained under the light of cognitive sciences, and they can have potential references to the neurosciences.

Keywords: Music, Visual arts, Mathematics, Gesture, Cognition.

1. Introduction and Current Research

Connections between the arts and sciences, in particular between music and physics/mathematics, date as far back to Pythagoras, and concern the relationship between sound, number, and proportion. Another great name, symbol of the dialogue among disciplines is Leonardo da Vinci, one of the protagonists among the Renaissance artists who were also architects, sculptors, literates, painters, and scientists. In several cases, arts gave an impulse to scientific development, as it happened for the Brunelleschi's perspective and, in more recent times, the birth of projective geometry (Wilczek, 2015). In the field of arts, music is the object of investigations under several points of sight:, such as mathematics, physics, psychology and cognitive sciences (Lerdhal and Jackendoff, 1983; Zbikowski, 2002; Mazzola et al., 2016b). Because music deals with sound, acoustics is an immediate scientific reference for this discipline and a fertile field of research (Castellengo, 2010).

The symbolic layer of music deserves a particular attention: there are several music theory studies that involve geometry to investigate the relationships between chords (Tymoczko, 2011), and the translation of symbolic parameters into physical ones (Mazzola et al., 2016–performance theory). In particular, to transform symbols into sounds, musicians are trained to perform specific movements, such as hand gestures as in piano or violin playing, hand and breathing skills for wind instruments, and also almost completely inner movements for singers, mainly involving diaphragm and larynx (Mannone, 2017a). Attention toward gestures in more qualitative terms is not new (Cadoz and Wanderley, 2000; Gødoy and Leman, 2010). Scholarly work also addresses the analogies between music and movement (Zbikowski, 2002; Eitan and Granot, 2006).

Mathematical theory of musical gestures, started with a pioneering work in 2007 (Mazzola and Andreatta, 2007) in the framework of category theory[1] (Mac Lane, 1971), is still under development (Mannone, 2017a, b), and it involves not only theoretical research but also creative applications in music composition. In a nutshell, gestures are described as curves in space and time. Recent developments in the mathematical theory of music can also be applied to musical performance and composition. At the light of gestures, techniques to translate music into images and vice versa through mathematics (Mannone, 2011) can be revised and improved (Mannone, 2017a). If we see a drawing as the result of a drawing gesture, we can envisage the same gestural matrix that can generate either a drawing or a sound. For example, the same 'staccato' gesture in conducting can generate a 'staccato' articulation in music, as well as, in visual arts, a series of points on canvas; the same applies for a more 'legato' gesture, or for a more energetic versus soft movement. From this, the idea of gestural similarity arises. In music, we may talk about gestural similarity if the gestural curves can be homotopically (i.e., continuously deformed) transformed the one into the other, and if the obtained sound spectra are similar. For example, we may have a 'vibrato' performed by a soprano, and a 'vibrato' performed by a flutist: there are analogies in the spectrum of both, and also in the movement they did: movements with the diaphragm (flutist) and the larynx (soprano). Ideally, the listener can recognize these analogies even if he or she is not a professional musician, and it should also be easy to recognize analogies between lines and music derived from them.

This topic is currently under research, but we can already cite several studies to support this idea, in the field of crossmodal correspondences (Roffler and Butler, 1968; Spence, 2011) between cognitive sciences and neurosciences (Bertolucci XX), between acoustics and linguistics (Nobile, 2013), between images and sounds (Uznadze, 1924; Köhler, 1929; Parise and Spence, 2012). In support of sonification of shapes, we may refer to the theory of indispensable attributes: the pitch (and loudness) for the auditory channel, and the shape for the visual channel (Kubovy and Schutz, 2010). We may imagine a sou source that is moving along a shape, and we would expect to hear the pitch (and loudness) changing accordingly. In fact, the most successful examples of sonification confirm crossmodal correspondences. We will give some examples of these ideas in the following section. More mathematical developments include applications of the theory of knots and braids, and the use of 2-categories to define more conveniently gestures and gestures of gestures (Mannone, 2017b).

2. Examples

In this section, we briefly cite some examples of creative applications of the ideas discussed in Section 1.

Correspondences between sounds and images are in itself a huge field of research and creation, as witnessed by Oskar Fischinger's and Iannis Xenakis' works. In particular, Xenakis invents the UPIC device to translate visual shapes into music, and he uses tridimensional graphs to investigate musical timbre (Xenakis, 1971). Salvatore Sciarrino, in his analysis of patterns and variational strategies in music and in visual arts, uses 2-dimensional graphical renditions to analyze musical scores: pitch and time are mapped to height and length respectively, different thickness indicates different levels of loudness, and the color the timbre (Sciarrino, 1998). Such a 2-dimensional representation is generalized by Marco Betta in 3-dimensions, and the conversion visual-to-sound is generalized in 3-dimensions by M. Mannone, where the third space-dimension is the loudness.[2] The most effective examples of conversions sound-to-visuals and vice versa verify crossmodal correspondences, see Section 1 for some references.

[1] In mathematics, a category is defined as a collection of points (objects) and arrows between them (morphisms). A category can itself be considered as a point, and arrows between categories are called *functors*. Associativity and identity properties must be verified. See Lawvere and Schanuel book (*Conceptual Mathematics*) for a basic and clear introduction.

[2] These examples are part of the huge field of sonification, that involves classical studies as well as contemporary applications. A recent journal article "A Brief History of Artists Turning Pictures into Music and Vice Versa" in the online review "Creators" cites several examples, included recent Andrew Huang's and Aleksander Vinter's works

Let us consider two recent applications, that also contain references to mathematical theory of musical gestures: the musical rendition of the shape of the water lily and the armor of the pangolin, a little animal with scales constituting an armor. In fact, a drawing can be seen as the result of a drawing gesture. We can have gestures combined with other gestures. For example, let us think of the shape of a leaf, and let us think of a line to represent the branch. We can combine both: drawing a branch with leaves, we are making a "gesture of gestures" (hypergesture). As another example, we may consider the shape of a water lily, and see how it can be musically rendered. The water lily is first drawn and then simplified into basic lines. The same melody has been associated to each of the large petals, imitating the shape of their borders, and another melody to each of the small petals. Each petal was indicated with a different timbre: brass, human voice, wind, string, percussion, and again brass. To give the idea of the conical shape and of the "movement alongside the shape," a change in pitch and loudness has been used: as in the Doppler effect, the closer are the petals, the louder and higher (of half-tone) is the melody. The "nymphaea" music belongs to an extended piece for voice and orchestra, see the recording at the webpage https://soundcloud.com/maria-mannone/mannone-genesis-of-music-from-gestures-second-part-fugue, with the water lily from 5'05'' to 6'20''.

In the second example, the pangolin, we start from a complex image: the closed armor of the defensive position of the animal. This image can be built starting from a single scale, that is repeated and then shifted; the envelope of the resulting image is modified (the scales are larger at the center and smaller at the sides), and finally closed in a loop. This can be described via a sequence of steps connected by arrows. Each step can be translated into music—a "scale" into a "musical scale," for example—obtaining a parallel sequence of musical variations (and we have arrows from visuals to sound, from visuals to visuals, and from sound to sound). We reduce the complexity to a collection of transformation of a simple element. This sequence of transformations can constitute the basis for composing, programming, and improvising. A recording of a free improvisation on this scheme is available: https://www.youtube.com/watch?v=uZvCxx5taho&list=PLnVUaUK-8Xnu3rvUcj-6tb3srKuh5ZW0m from 15'02''. In this last example, we also applied concepts of visual networks (Guibas 2014) and sound-image networks, framed in category theory (Mannone, 2017b).

3. Conclusion

We summarized some main points in the mathematical theory of musical gestures, connecting them with the topic of sonification, giving some examples of creative applications. Further developments of these ideas may involve perception experiments to investigate the analogies between visual shapes and the music derived from them. Former experiments proved the influence of the vision, and of the vision of gestures, in the perception of music (Schutz and Lipscomb, 2007).

This research may be framed in the field of interdisciplinary applications of category theory. In this frame, we can see mathematics as a philosophical discipline in itself, that gives us a different perspective on arts, and on the interaction between different arts.

Category theory has already been successfully applied to several sciences (Spivak, 2014), and it is relevant as a unifying approach. Diagrammatic thinking is also relevant in philosophy (Alunni, 2004). According to Patras (2001), this, however, does not mean that category theory *is* more important than other theories; it constitutes a relevant point of sight—not the only one. For musical purposes, and to connect disciplines, the categorical language appears clear and, at least for some basic examples, intuitive also for non-mathematicians. Patras also highlights the importance of mathematics as a philosophical

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on 2-dimensional images and MIDI sound files (Pangburn, 2017) The same article cites Mannone's Eiffel Tower: "For the score of 'Eiffel Tower,' [they were] selected 900 pixel-coordinates of one side of the building, transferred them into Mathematica software, created the MIDI file, and finally exported it into Sibelius [notational software] and listened to it."

discipline in itself, and the necessity of clear explanations and teaching for students (Patras, 2001). Arts applications may help learning: the more sensorial channels we may use to approach abstract ideas, the better could be our understanding and our ability to share ideas.

Other applications of these research topics should involve teaching strategies and pedagogy: a new approach to teach music to scientists, to teach science to musicians, and to teach music and science together, through a clear language, with intuitive examples. In fact, one of the challenges of interdisciplinary studies is the language and the connecting language between disciplines. Let us imagine disciplines as islands, with a building edified on each of them.

The level of the knowledge in each field may be compared with the height of the building: the more we know, the higher is the building. However, if we would like to connect islands, it is easier and more effective to build bridges near to ground-level of each island, instead of connecting the tops of their buildings. It means that it is easier and more effective to connect disciplines through their basic ideas and simple language, instead of connecting between them complex and deep concepts full of jargon. A more simple language allows a more easy access to interdisciplinary studies between mathematics, music, physics, and visual arts. The scope is to reach a broader and broader public, stimulating the interest and the curiosity of people.

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