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USING QUALITY CRITERIA TO EVALUATE STUDENTS' PERCEPTIONS OF GEOGEBRA IN TEACHER EDUCATION

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GeoGebra is considered as one of the most important digital tools in educational settings ranging from school to university level. GeoGebra is an open-source software that is available anytime and anyplace. The tool has a wide range of functionalities, and it has been constantly improved to make the user interface more user-friendly, and the mathematical activities more engaging for students and teachers. But still, despite its learning potentialities, GeoGebra has not been fully evaluated to assess its usefulness in teacher education. This work aims at using a set of quality criteria to evaluate the value of GeoGebra in teacher education.

Keywords: GeoGebra, Pedagogical usability, Quality criteria, Technical usability.

Introduction

GeoGebra is considered as one of the most important digital tools in mathematics education for teacher students. It is a potentially powerful tool for problem solving, simulation, and modelling. It can be used for algebraic manipulation, to draw geometric figures and graphs, analyze functions, present statistics, use simulations, evaluate, and assess the results produced by the tool. However, despite great interest in the tool and the promising results achieved so far, research work remains to be done to fully exploit the potentialities of GeoGebra. The educational value of GeoGebra has not yet been fully evaluated according to quality criteria that are relevant in educational settings. The purpose of this paper is to use an evaluation instrument to assess the value of GeoGebra for teaching and learning mathematics. The paper also addresses the implications and supporting conditions that help placing GeoGebra as an integral tool in mathematics education.

GeoGebra

GeoGebra is a free, open-source, and multi-platform dynamic mathematics software that combines geometry, algebra, and calculus. It has also a CAS and spreadsheet function (http://www.geogebra.org). Geogebra allows students to create geometrical figures, manipulate algebraic equations, and investigate their properties, solve equations, graph functions, create constructions, analyze data, and explore 3D mathematics (Figure 1).

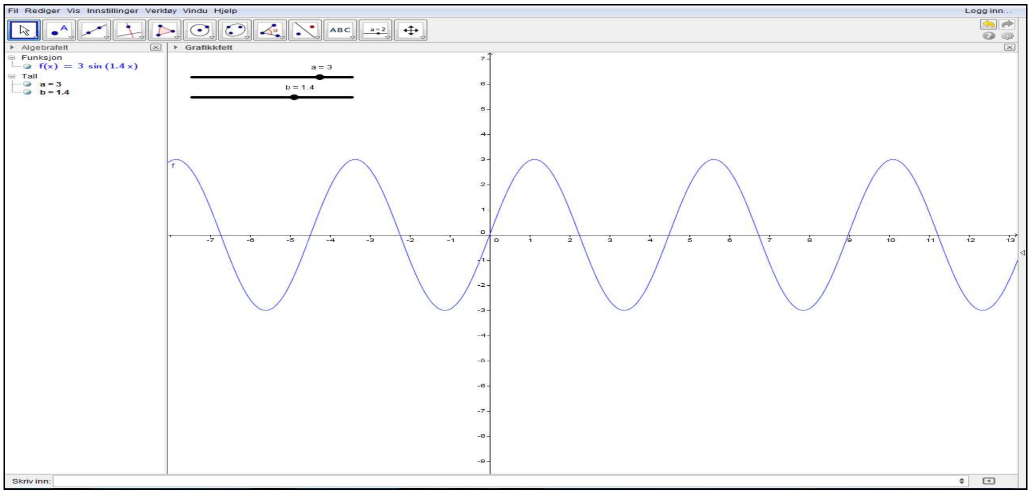

Figure 1. A screenshot of GeoGebra showing the function f(x) = 3*sin(1.4x)

GeoGebra covers the whole mathematics subject curriculum, and it can be used from elementary school to university level. According to Hohenwarter and Jones (2007), the most notable feature of GeoGebra is that it offers two representations of every object: Every expression in the algebra window corresponds to an object in the geometry window and vice versa providing the possibility to move between the algebra and the geometry window, and, as a result, providing a deeper insight in the relations between geometry and algebra. On the one hand, the geometric representation can be modified by dragging it with the mouse like in any other dynamic geometry system, whereby the algebraic representation is changed dynamically. On the other hand, the algebraic representation can be changed using the keyboard causing GeoGebra to automatically adjust the related geometric representation. As a result, GeoGebra is a dynamic geometry software that allows the simultaneous visualization of two, or even three views (graphic, algebraic and tabular) of the same mathematical objects. This possibility allows a user to observe the variations taking place in every kind of illustration when the user makes any changes.

Literature Review

There is an enormous interest in digital tools in mathematics education for teacher students. Textbooks and Web sites are filled with pictures, figures, diagrams, and graphs, and diverse tasks. More specifically, there are several research studies on GeoGebra and its impact on students' learning of mathematics (Dockendorff, & Solar, 2018; Fahlberg-Stojanovska & Stojanovski, 2009; Gómez-Chacón, 2011; Haciomeroglu, 2011; Hall & Chamblee, 2013; Little, 2011; Mousoulides, 2011; Zengin, 2018, 2017). Despite the potentialities of GeoGebra to foster mathematical learning, there are constraints in terms of technical, pedagogical, and mathematical use of the tool. More specifically, Doruk, Atuemen, and Aytekin (2013) indicate that it is difficult to transcribe mathematical expressions into GeoGebra, because the tool is very sensitive, and it is not adequate in teaching basic mathematical concepts. Other problems are related to rotation of figures and inadequacy for 3D work (ibid). Likewise, Hall and Chamblee (2013) reveal that there still are problems to solve before GeoGebra can become widespread in mathematics education, such as adequate training. As a result, users may still be confronted with technical problems while using GeoGebra in learning mathematics. Thus, technical usability issues are still important for novice users, and that it takes time to learn it (Doruk, Atuemen, & Aytekin, 2013). Exploiting the learning potentialities of GeoGebra may be demanding in terms of efforts, making the process of instrumental genesis more complex (Trouche, 2004). Students need to be guided by the teacher's instrumental orchestrations (Guin & Trouche, 2002) in using GeoGebra to give the opportunity to tackle the constraints they might encounter (Faggiano & Ronchi, 2011). According to Kllogjeri (2010), GeoGebra should provide a link between mathematical textbooks and the tool for problem-solving situations. Furthermore, engaging students with motivating tasks is still important in mathematics. Hence, traditional mathematics cannot be underestimated, because the initial enthusiasm and excitement of using GeoGebra disappears over time, and it can be boring to use the tool for a long time (Doruk, Atuemen, & Aytekin, 2013).

Methods

This work is a single case study that was conducted at the university level in the context of a technology-based mathematics course for teacher students. The course provides theoretical background and insights from research on the use of digital tools in the learning and teaching of mathematics, and practical experience in the use of selected digital tools that are relevant to mathematics education. GeoGebra is one of the digital tools being used in the course. After a general presentation of the tool, the students (*N=8*) used it over a period of four weeks to discover its potentialities and constraints solving diverse mathematical exercises. After four weeks of exposure, the students were asked to assess GeoGebra using an evaluation instrument based on a survey questionnaire with more than 60 items that are categorized in 4 groups of predefined criteria (Technical usability, pedagogical usability, mathematical thinking, and assessment). To measure students' responses, the survey used a five-point Likert scale from 1 to 5, where 1 was coded as the highest and 5 as the lowest (1 = "Strongly Agree"; 2 = "Agree"; 3 = "Neither Agree or Disagree"; 4 = "Disagree"; 5= "Strongly Disagree"). In addition, the students were asked to provide feedback in their own words to open-ended questions.

Evaluation Criteria

The notion of usability is used to address the potentialities and constraints of digital tools in mathematics education. The research literature provides two definitions of usability: Technical and pedagogical usability. The former is a self-evident requirement for any digital tool (Nielsen, 1993). It measures the extent to which a digital tool is practicable for users. In many cases, however, the impact of technical usability on learning may be limited when it comes to pedagogical use of the tool. In addition, the functionality of a specific digital tool does not always result in pedagogical opportunities and gains (Burden & Atkinson, 2008). This is the case of tools that are designed without pedagogical goals, e.g., Spreadsheets. Following this line of argumentation, Nokelainen (2006) reformulated the notion of usability to include pedagogical usability criteria, such as learner control, learning activity, collaborative learning, goal orientation, applicability, added value, motivation, previous knowledge, flexibility, and feedback. Technical usability involves techniques for ensuring a trouble-free interaction with the tool while pedagogical usability aims at supporting the learning process. The goal of technical usability is to minimize the cognitive load resulting from the interaction with the tool in order to free more resources for the learning process (Hadjerrouit, 2010).

In addition, the interactivity criterion has emerged as one of the most important criteria for evaluating the quality of digital tools. Interactivity is associated with the feedback the tool provides to users. Feedback builds the ground both for formative and summative assessment. Feedback that supports formative assessment can be provided in terms of several assessment modes. It can take many forms, for example immediate feedback to students' actions. It can also provide a combination of conceptual, procedural, and corrective information to the students (Bokhove & Drijvers, 2010). Another way of providing feedback in terms of formative assessment is in the form of question types. Summative assessment is important for testing and grading, and it can be provided in form of statistics, data logs, and protocols. Another important criterion that needs to be considered is mathematical thinking in the design and use of the tool. Mathematical thinking includes conceptual understanding in terms of comprehension of mathematical concepts and symbols, operations, and relationships, and procedural skills in terms of flexibility, accuracy, and efficiency in manipulating mathematical concepts and figures (Bokhove &

Drijvers, 2010). GeoGebra should ensure a good representation of algebraic and geometric concepts, symbols, operations, and formulas. Likewise, another important criterion is the congruence between the tool features and paper-pencil techniques to capture mathematical symbols and concepts, and procedural skills as well. As a result, the criteria for evaluating GeoGebra are grouped in 4 main categories:

- Technical usability (ease-of-use, accessibility, management facilities)
- Pedagogical usability (motivation, variation, student autonomy, individualization, differentiation, and collaboration)
- Mathematical thinking (mathematical content, congruence with paper-pencil- techniques, and concretization of curriculum)
- Assessment (Feedback, formative and summative assessment)

These criteria are intimately related to each other. Technical usability is a prerequisite for making the tool pedagogically usable. Likewise, feedback and elements of formative and summative assessments are of particular importance for the learning process. The quality of the tool is also dependent on the extent to which mathematical thinking is integrated into the tool and congruence between the tool features and paper-and-pencil techniques.

Technical Usability

- Ease-of-use: *The tool should be easy to use, to start, and to exit.*
- Accessibility: *The tool should be accessible anytime and place.*
- Management facilities: *The tool should provide management facilities, for example it should be possible to store the answer given by the students, add and modify content (questions, texts, feedback). The tool should also have readily available content.*

Pedagogical Usability

- Motivation: *Motivation measures the extent to which GeoGebra is attractive to use, adapted to the students' age, knowledge level, development, and interests, as well as tied to the students' other activities and tasks. Using GeoGebra should be a motivational factor for learning mathematics.*
- Variation: *GeoGebra should be able to present the content in several ways, and facilitate various activities with students, and used in combination with textbooks. GeoGebra should also be used as an alternative to achieve variation in teaching.*
- Student autonomy: *GeoGebra should enable a high degree of autonomy so that the students do not necessarily need teacher assistance and ask fellow students for help or use textbooks. The knowledge provided by GeoGebra should be potentially powerful to enable students to become less dependent on teacher assistance.*
- Individualization: *GeoGebra should take into account adapted education, and different knowledge levels. The students should be able to work at their own pace, save their work and continue later. Disabled people should be able to use the tool.*
- Differentiation: *Differentiation means that the tool should provide multiple tasks with different levels of difficulty and can be tailored to the students. In addition, the tool should provide opportunities for the teacher to make individual adjustments and customize the tool when needed.*
- Cooperation: *Cooperation is about the possibility for the tool to stimulate cooperation. It may also mean that the tool contains collaborative tasks, as well as communication tools are integral part of the tool.*

Mathematical Thinking

- Mathematical content: *Mathematical content (concepts, exercises, and problems) provided by GeoGebra should be of high quality. The content should be mathematically sound and faithful to the underlying mathematical properties. GeoGebra should also provide opportunities to display formulas correctly and help students to gain knowledge that is otherwise difficult to acquire.*
- Congruence with GeoGebra techniques: *GeoGebra should enable the student to apply his/her own paper-and-pencil technique reasoning steps and strategies, and express mathematical ideas, as well as facilitate students' mathematical activities.*
- Concretization of curriculum: *This criterion measures the extent to which the tool provides opportunities to concretize the curriculum, and whether the tool is tied to teaching and adapted to the curriculum.*

Assessment

- *Formative assessment* is an important criterion to measure the extent to which GeoGebra provides several assessment and review modes, appropriate feedback in the process of problem solving, and use of several question types. Providing student profiles may also be important when it comes to adapt the questions to the student's knowledge level. In terms of *summative assessment*, the tool should provide teachers with quantitative data, statistics and results that help to evaluate students' performances. It should give scores and grading.

Results

***Technical Usability* (Table 1)**

Globally, students agreed that GeoGebra is easy to use, to start, and to exit (MEAN=2.57). Likewise, GeoGebra stores the answers given by the students (MEAN=2.14), and the students' solution process (MEAN=2.00). In contrast, it does not have question management facilities and content, as well as a function to add and modify content (MEAN=3.43). Most students were satisfied with the accessibility of GeoGebra in terms of location and time (MEAN=2.57). GeoGebra is a free-software that is accessible anytime and anywhere.

Table 1. Technical usability

	Mean	**Std. Dev**
Geogebra is easy of use, to start and to exit	2.57	0.787
Geogebra stores the answers given by the students	2.14	1.069
GeoGebra stores the students' solution process	2.00	1.528
GeoGebra has management facilities	3.43	0.976
GeoGebra is accessible at any place and anytime	2.57	0.787

Pedagogical Usability

***Motivation* (Table 2)**

Concerning motivation, the majority of the students agreed that GeoGebra is adapted to the students' age, knowledge level, development and interests (MEAN=2.14). Likewise, GeoGebra can motivate students to use it in mathematical classroom contexts (MEAN=1.86). Otherwise, the students were more spilt in their views regarding the attractiveness, visual design, and appearance of the tool (MEAN=2.86), or that the tool is attractive for the user (MEAN=2.43). Likewise, the majority of the students believed that the tool is tied to the student's other mathematical activities (MEAN=2.43).

Table 2. Motivation

	Mean	Std. Dev
Geogebra has a good visual design and appearance	2.86	0.690
Geogebra has an attractive appealing for users	2.43	0.976
GeoGebra is adapted to students' knowledge level	2.14	0.378
GeoGebra is tied to students' other mathematical activities	2.43	1.272
GeoGebra motivates students to do mathematics	1.86	0.690

Student Autonomy (Table 3)

More than half of the students did perceive that the tool enables a high degree of student autonomy (MEAN=2.00). This means that students still need to ask their teacher (MEAN=3.57) and fellow students for help and assistance (MEAN= 3.00). Furthermore, the majority felt that GeoGebra cannot fully replace textbooks as a resource to support the solving of mathematical problems and tasks (MEAN=2.57). As a result, GeoGebra enables a certain degree of student autonomy, but the problem-solving process needs to be controlled by the teacher to a certain degree (MEAN=3.29).

Table 3. Student autonomy

	Mean	Std. Dev
GeoGebra enables a high degree of autonomy	2.00	0.577
Students need to ask their teacher for help	3.57	0.787
Students need to ask fellow students for help	3.00	0.816
Geogebra can replace textbooks	2.57	0.787
The problem solving is controlled by teacher	3.29	0.488

Variation (Table 4)

The vast majority of the students believed that the tool can be used as an alternative to achieve variation in teaching (MEAN=1.29), and that the tool can be used in combination with textbooks as supplementary aid (MEAN=1.57). Likewise, the majority believe that GeoGebra facilitates various activities (MEAN=2.00). In contrast, most students did not think that GeoGebra presents the algebraic content in several ways (mathematical, graphical, and textual) (MEAN=2.71).

Table 4. Variation

	Mean	Std. Dev
Geogebra can be used as alternative to achieve variation	1.29	0.488
Geogebra presents the content in several ways	2.71	0.756
GeoGebra facilitates various activities	2.00	0.816
GeoGebra can be used in combination with textbooks	1.57	0.535

Individualization (Table 5)

The results reveal that more than half of the students think that GeoGebra takes into account the requirement of adapted education (MEAN=2.14), and that it is possible and easy to adapt the tool to different students' knowledge levels (MEAN=1.86) and provide different challenges (MEAN=1.57). For most students, it is also possible to save their work and continue at a later time (MEAN=1.29). Most students also think that the tool enables students to work at their own pace (MEAN=1.43). In some contrast, the majority think that GeoGebra facilitates disabled people to work with (MEAN=2.43).

Table 5. Individualization

	Mean	**Std. Dev**
GeoGebra takes into account the requirement of adapted education	2.14	0.900
it is easy to adapt GeoGebra to different students' knowledge levels	1.86	0.378
GeoGebra facilitates disabled people to work with	2.43	0.535
GeoGebra provides different challenges	1.57	0.535
GeoGebra enables students to work at their own pace	1.43	0.535
GeoGebra helps to save the work and continue at a later time	1.29	0.488

Differentiation (Table 6)

Less than majority think that the tool contains multiple levels of difficulty and may be tailored to the students (MEAN=2.71), and that both teachers and students can customize the tool when needed (MEAN=2.71). In addition, less than the majority believe that the tool is designed to help students choose the level of difficulty (MEAN=3.00). Likewise, less than the majority think that the tool provides opportunities for teacher to make individual adjustments for each student and define new exercises (MEAN=3.00). Similarly, less than the majority think that GeoGebra is beforehand adapted to special student groups (age, level of ability) (MEAN=3.14). As a result, differentiation is not met.

Table 6. Differentiation

	Mean	**Std. Dev**
GeoGebra has multiple levels of difficulty, and can be tailored to the students	2.71	1.380
Both teachers and students can customize GeoGebra	2.71	1.380
GeoGebra is designed to help students choose the level of difficulty	3.00	1.000
GeoGebra is beforehand adapted to special student groups	3.14	1.215
Geogebra provides opportunities for teacher to make individual adjustments	3.00	1.155

Cooperation (Table 7)

Regarding cooperation, most students disagree that GeoGebra stimulates students to cooperate (MEAN=3.00), mostly because communication and collaboration tools are not integrated into GeoGebra (MEAN=3.71). Likewise, most students think that the tool does not contain collaborative and group tasks and exercises (MEAN=3.86).

Table 7. Cooperation

	Mean	Std. Dev
GeoGebra stimulates students to cooperate	3.00	0.816
Communication and collaboration tools are not integrated into GeoGebra	3.71	1.113
GeoGebra contains collaborative and group tasks and exercises	3.86	0.900

Mathematical Thinking

Mathematical Content (Table 8)

While most students globally agree that GeoGebra has a high quality of mathematical content (MEAN=1.29), the exercises are not well-designed and formulated (MEAN=3.00). Likewise, the majority think that the tool enables students to express mathematical ideas and strategies, or that GeoGebra facilitates students' mathematical activities (MEAN=2.14). Furthermore, the majority believe that the tool is mathematically sound and faithful to the underlying mathematical properties, e.g., conventional representations of mathematical expressions, and sound operations (MEAN=2.57), and in lesser degree, the tool is able to display mathematical formulas correctly (MEAN=3.00). In some contrast, more than half of the students think that the tool provides opportunity to help students train and gain knowledge that is otherwise difficult to acquire (MEAN=2.29).

Table 8. Mathematical content

	Mean	Std. Dev
Geogebra has a high quality of mathematical content	1.29	0.488
Geogebra exercises well designed and formulated	3.00	1.155
Geogebra is mathematically sound and faithful	2.57	1.134
Geogebra helps to gain and train knowledge that is difficult to acquire	2.29	0.488
Geogebra displays mathematical formula correctly	3.00	0.816
Geogebra facilitates mathematical activities	2.43	0.787

Congruence with Paper and Pencil Techniques

In addition to the mathematical content, most students do not fully agree that GeoGebra enables them to apply their own paper and pencil reasoning steps and strategies (MEAN=3.00).

Concretization of the Curriculum (Table 9)

Concerning this issue, most students agree that the tool is appropriate to its objectives and students' level or can be adapted to the curriculum (MEAN=1.57). Likewise, most students agree that the tool is tied to teaching (MEAN=1.86). Otherwise, most students think that the tool provides opportunities to concretize the curriculum (MEAN=1.14) or enables the teacher to concretize the curriculum (MEAN=1.71).

Table 9. Concretization of the curriculum

	Mean	Std. Dev
Geogebra can be adapted to the curriculum	1.57	0.535
GeoGebra provides opportunities to concretize the curriculum	1.14	0.378
GeoGebra is tied to teaching	1.86	1.069
GeoGebra enables the teacher to concretize the curriculum	1.71	0.488

Assessment (Table 10)

Most students do not agree that GeoGebra provides several assessment modes, e.g., practice, test, observation (MEAN=3.43). In addition, most students agree that the tool is not able to check a student's answer (MEAN=4.43). Likewise, most students think that the tool does not give directly feedback to the students in the process of problem solving (MEAN=3.86). The tool does not provide the teacher with statistics and results of students' problem-solving process (MEAN=4.14). Moreover, most students disagree that the tool caters for several types of feedback (conceptual, procedural, corrective) (MEAN=4.29). Likewise, the tool does neither take the mastery and profile of the student into account nor serve up appropriate questions (MEAN=3.57). Furthermore, GeoGebra does neither show what the student has done wrong or wright (MEAN=4.29). In addition, most students agree that the tool does not allow for the use of several question types, e.g. multiple choice (MEAN=4-14). Finally, almost all students agree that the feedback does not contain motivating elements and sound effects (MEAN=3.86).

Table 10. Assessment

	Mean	**Std. Dev**
GeoGebra provides several assessment modes	3.43	1.272
GeoGebra caters for several types of feedback	4.29	0.756
GeoGebra takes the mastery and student profile into account	3.57	0.976
GeoGebra show what the student has done wrong or wright	4.29	0.951
GeoGebra allows for the use of several question types,	4.14	1.215
GeoGebra is able to check a student's answer	4.43	0.787
GeoGebra gives directly feedback to the students	3.86	0.690
GeoGebra provides the teacher with statistics	4.14	0.900
GeoGebra contains motivating elements and sound effects	3.86	0.900

Discussion and Implications

The *first issue* raised in this study is the identification of evaluation criteria that are pertinent to GeoGebra. These include technical and pedagogical usability criteria, which are self-evident requirements for any digital tool in educational settings. Likewise, digital tools should support mathematical thinking and reflection. Another important category is feedback in terms of formative and summative assessment. These four broad categories of criteria provide insight in the various characteristics of GeoGebra and help to capture and assess the potentialities and constraints of GeoGebra in educational settings. The *second issue* reported in this study refers to students' perceptions of GeoGebra based on the evaluation criteria. A survey questionnaire was designed, and the students were asked to evaluate GeoGebra using the criteria. The *third issue* of this study relates to the extent to which the criteria are met.

In this section, the results are discussed, and some implications are drawn for the learning of mathematics. From the technical usability point of view, the students were globally satisfied with GeoGebra in terms of ease-of-use, user-friendly interface, availability of mathematical content, and accessibility (Table 1). The negative point with GeoGebra is that it does not have management facilities, as well as a function to easily add and modify the mathematical content. But, there exit Web portals where GeoGebra files can be uploaded. Despite this limitation, GeoGebra is relaltively easy to use, which is a prerequisite for using the tool to achieve a pedagogical goal.

In terms of pedagogical usability, many issues have been addressed: Motivation, autonomy, individualization, differentiation, variation, and cooperation. Regarding motivation, students pointed out that GeoGebra does not have an attractive design, mainly because it does not contain informative elements in form of motivating animations or sound effects, which may be particularly important in school education (Table 2). Thus, it may be useful to integrate multimedia elements in the design of the tool to make the symbolic and abstract representation of mathematical concepts more appealing, at least at the school level.

In addition, enhanced motivation can be achieved through realistic mathematical tasks, animations, and dynamic simulations and visualizations, and a combination of these (Pierce, & Stacey, 2011). Despite this constraint, most students think that the tool is adapted to their age and development level, which is a motivational factor in keeping them engaged in mathematics. Depending on the didactical purpose, GeoGebra seems to provide motivating tasks that appeal to teacher students, and this point is to some extent revealed in this study.

Beyond motivational issues, many students think that GeoGebra enables a high degree of student autonomy allowing them to work at their own pace, but not to work independently from teacher assistance (Table 3). Likewise, textbooks are still needed when using GeoGebra in classroom, including online study material. Moreover, applications by means of mobile technology open the door for teacher students, and let the learning of mathematics occurs anytime, anywhere and in a variety of contexts. Nevertheless, traditional ways of doing mathematics with paper and pencil are still important to stimulate learning.

Moreover, students think that GeoGebra can be used as an alternative to achieve variation in teaching. The tool also facilitates various activities, and it can be used in combination with textbooks (Table 4). This is in line with the research literature that indicates that variation in teaching is important because students learn in different ways (Hadjerrouit, 2017). New applications like GeoGebra graphic, geometry, and 3D calculator may contribute to variation. Furthermore, individualization is globally achieved for most students (Table 5), except for disabled people to work with. However, differentiation is not sufficiently realized (Table 6). For example, GeoGebra does not allow users to customize the tool when needed or adapt it beforehand to special student groups. Here, the role of the teacher and instrumental orchestration of students' instrumental genesis cannot be underestimated.

The results also indicate that GeoGebra does not stimulate students to cooperate, partly because communication tools are missing. Furthermore, GeoGebra does not contain group tasks (Table 7). Despite this limitation, cooperation possibilities are given by the community of researchers on the Web (Hohenwarter & Lavicza, 2009). Even though discussions happened in classroom, it appears that GeoGebra does not contribute much to interactions in classroom. It would be important in future research to examine how and whether students work together in small groups when solving GeoGebra exercises.

In terms of assessment issues, GeoGebra does not fulfill the criteria listed in Table 10. Bokhove and Drijvers (2010) argues that digital tools should provide formative feedback to the work students are doing, e.g., in the form of review modes, because this type of feedback supports the learning process. However, GeoGebra lacks this kind of feedback on the mathematical work being performed. GeoGebra cannot assess what the student has done wrong or right and cannot record students' solution strategies. Moreover, GeoGebra does neither build student profiles nor serve up appropriate questions to the students. The lack of student profiles may prevent users from engaging in mathematical activities, particularly when the student does not know a topic well enough. Adaptability of GeoGebra can be achieved by considering different learning styles and increased differentiation. Finally, it is possible that the students had found GeoGebra more useful if it has provided formative feedback so that they can follow the solution process step by step. On the other hand, dynamic visualizations give another type of feedback by showing mathematical concepts dynamically. This is the strength of GeoGebra. Indeed, being able to visualize mathematical problems helps to create more sense of problem solving and promote conceptual understanding. Hence, the lack of feedback is compensated by dynamic visualizations.

Despite these limitations, the strength of GeoGebra is the high quality of mathematical content, and the potentialities it provides to support students, and help them gain knowledge that is otherwise difficult to acquire (Table 8). In this context, Diković (2009) found that students using GeoGebra achieved a better understanding of mathematics compared to similar tools. GeoGebra gave students an intuitive feel of mathematical concepts. Moreover, the tool allowed them to explore a wide range of function types and gave them the opportunity to create links between symbolic and visual representations of mathematics.

In addition, the tool's openness enables students to express mathematical ideas and strategies. The negative side is that the tool is not fully congruent with paper-pencil techniques, since GeoGebra notations are not fully similar to mathematical representations. Furthermore, it does not fully display formulas correctly as it is not fully mathematically sound to the underlying properties of mathematics. Clearly, doing mathematics with GeoGebra cannot be successful, unless it can be proved what is happening around for example the number "zero" (e.g. division by zero), because of the limitations of computers when dealing with such numbers (Antohe, 2009). Hence, only a sound pedagogy can connect the positive and negative sides of both "traditional mathematics" and "machine mathematics". In addition, research is needed to understand the co-emergence of GeoGebra and paper-and-pencil techniques in order "to promote argumentation abilities" in geometry (Iranzo & Fortuny, 2011, p. 102). It is also necessary to understand the role of the teacher in orchestrating students' instrumental genesis when using GeoGebra (Trouche, 2004).

Finally, from the curriculum point of view, most students think that GeoGebra enables the teacher to concretize the mathematics subject curriculum, and that the tool is tied to teaching mathematics. Likewise, GeoGebra can easily be adapted to the curriculum (Table 9).

Conclusion

After four weeks of using GeoGebra in teacher education, empirical data have been collected and analysed using a survey questionnaire and evaluation criteria. Even though, it has been possible to make some reasonable interpretations of the results, it is difficult to conclude direct causality between students' experiences with GeoGebra and the learning of mathematics, since several contextual factors may implicitly or explicitly affect the learning process in authentic educational settings. The sample size is also limited. Nevertheless, the results indicate that GeoGebra shows potential for learning mathematics, although not all criteria are equally met, such as assessment, differentiation, and cooperation. To gain more insight in the potentialities and constraints of GeoGebra, the study will be repeated to achieve more reliability and validity.

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