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IMPACTS OF VISUALIZATION TOOLS ON MATHEMATICAL LEARNING IN TEACHER EDUCATION: A CRITICAL EVALUATION

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Introduction

This paper aims at evaluating students' engagement with mathematics in a digital environment. It is a continuation of the previous projects that were carried out in 2016 and 2017 (Hadjerrouit, 2015, 2017). More specifically, the article focuses on the affordances and constraints of visualization tools and students' learning of mathematics using SimReal. The concept of affordances is used as a theoretical lens to collect and analyse empirical data. SimReal is a visualization tool for teaching and learning mathematics for a wide range of topics. It uses a graphical calculator, video lessons and simulations, live streaming, and interactive simulations. SimReal has more than 5000 applications, exercises, and tasks in various areas of mathematics (Brekke, & Hogstad, 2011). The tool has been constantly improved to make the interface more user-friendly, and mathematical activities more motivating and engaging for students. The reason for using SimReal is that there is a huge interest in visualization tools in mathematics education, but there are few empirical studies in teacher education (McKenzie, & Clements, 2014; Macnab et al, 2012; Presmeg, 2012).

Theoretical background: The Concept of Affordance

The concept of affordance is used as a theoretical lens to explore the extent to which SimReal affords students' engagement with mathematics (Hadjerrouit, 2017). The concept of affordance proposed by James J. Gibson in his book "*The Ecological Approach to Visual Perception"* (Gibson, 1977), refers to the relationship between an object's physical properties and the characteristics of a user that enables particular interactions between user and object. More specifically, Gibson used the term "affordance" to describe the action possibilities offered to an animal by the environment with reference to the animal's action capabilities.

The concept of affordances was introduced to the Human-Computer-Interaction community by Donald Norman in his book "*The Psychology of Everyday Things"* (Norman, 1988). Accordingly, affordances refer to the perceived and actual properties of the thing (physical object, computer, etc.), primarily those fundamental properties that determine just how the thing or object could possibly be used.

Several research studies used Norman's ideas to implement the concept of affordances in various educational settings. For example, Turner and Turner (2002) specified a three-layer articulation of affordances: Perceived affordances, ergonomic affordances, and cultural affordances. Likewise, Kirchner, Strijbos, Kreijns, and Beers (2004) described a three-layer definition of affordance:

Technological affordances that cover usability issues, educational affordances to facilitate teaching and learning, and, social affordances to foster social interactions. In mathematics education, Chiappini (2012) applied the notions of perceived, ergonomic, and cultural affordances to Alnuset, a digital tool for high school algebra.

Based on these research studies, this paper proposes a theoretical lens based on three types of affordances at six different levels for SimReal or similar tools (Fig.1):

a) Technological affordances that describe the technicalities and functionalities of the tool (Level 1)
b) Pedagogical affordances:
   - Pedagogical affordances at the student level or mathematical task level (Level 2)
   - Pedagogical affordances at the classroom level or student-teacher interaction level (Level 3)
   - Pedagogical affordances at the subject level, that is the area of mathematics being taught (Level 4)
   - Pedagogical affordances at the assessment level (Level 5)
c) Socio-cultural affordances that cover curricular, cultural, and ethical issues (Level 6)

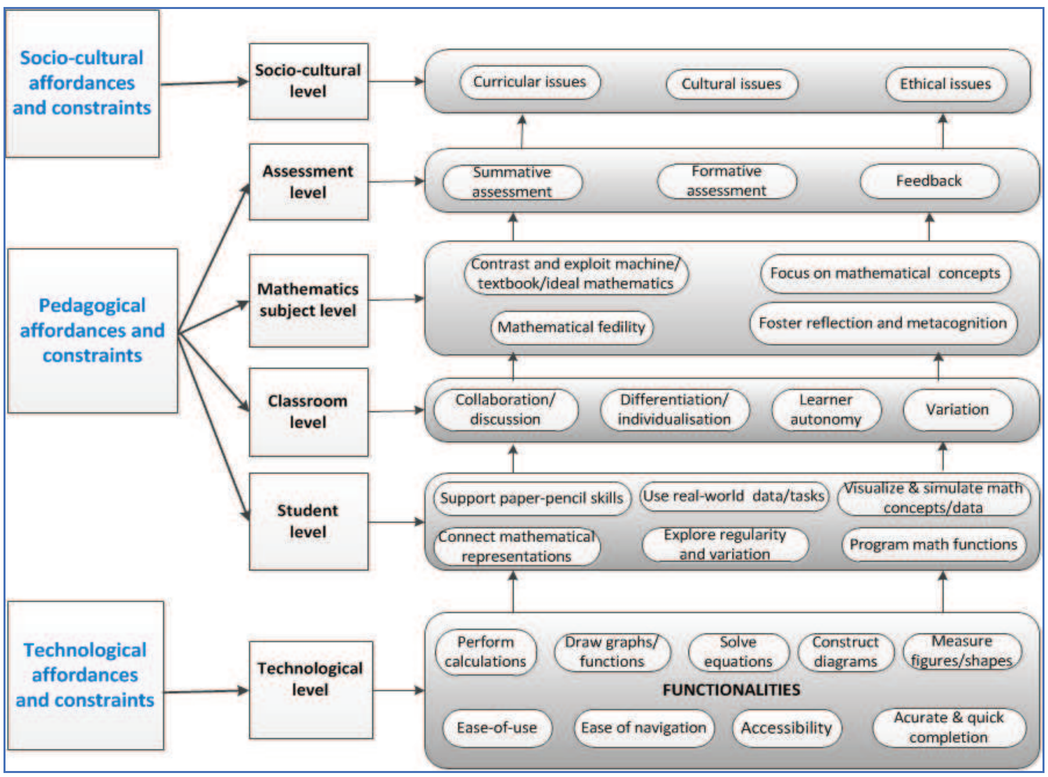

Figure 1. Affordances of SimReal (Hadjerrouit, 2019)

Research Methods

Twenty-two teacher students (*N=22*) from a technology-based course in mathematics education participated in this project. The students were categorized on the basis of their study programmes: Primary teacher education level 1-7, primary teacher education level 5-10, advanced teacher education level 8-13, and mathematics master's programme. The recommended pre-requisites were basic knowledge of ICT (Information and Communication Technology) and experience with standard digital tools like text processing, spreadsheets, calculators and Internet. No prior experience in SimReal was required.

A digital learning environment centred around SimReal over three weeks was created, starting from 21 August to 8 September 2017. The environment included video lectures, visualizations, and simulations of basic, elementary, and advanced mathematics, and diverse online teaching material. Basic mathematics focused on games, dices, tower of Hanoi, and prison. Elementary mathematics consisted of multiplication, algebra, Pythagoras' theorem, Square theorem, and reflection. The topics of advanced mathematics were measurement, trigonometry, conic section, parameter, differentiation, and Fourier. Both quantitative and qualitative methods were used to collect and analyse students' engagement with mathematics using SimReal. The following methods were used:

1) A survey questionnaire with a five-point Likert scale from 1 to 5
2) Students' written comments to each of the statements of the survey questionnaire
3) Students' written answers to open-ended questions
4) Analysis of students' comments from method 2 and answers to open-ended questions from method 3
5) Task-based questions on Pythagoras' and Square theorem

The data collection and analysis methods were guided by the concept of affordances, and some open-coding to bring to the fore data that is not covered by the theoretical lens. This paper focuses on the results gained by method 1, and a detailed presentation and analysis of the results achieved by method 5. The results by means of the other methods will be presented in another article with a more a qualitative approach to data analysis, that is students' written comments to each of the statements of the survey questionnaire, students' written answers to open-ended questions, and analysis of students' comments from method 2 and answers to open-ended questions from method 3.

Results

Technological, Pedagogical, and Socio-cultural affordances

These are the results obtained mostly by means of the survey questionnaire (method 1), which intended to collect data on affordances and constraints of SimReal.

The results obtained by means of the survey questionnaire can be summarized as follows. In terms of technological affordances, SimReal has a ready-made mathematical content at all levels of mathematics from elementary to advanced topics. Furthermore, SimReal video lessons, simulations, animations, and live streaming are of good quality. In terms of constraints, the overwhelming majority felt that SimReal still lacks a user-friendly and intuitive interface.

In terms of pedagogical affordances, four levels can be distinguished. Firstly, at the student level, SimReal provides motivating real-world mathematical tasks at all levels of the study programmes. It also facilitates various activities (problem-solving, video lectures, live streaming), and several ways of representing mathematical knowledge (texts, graphs, symbols, animations, visualizations). The combination of live streaming, video lectures, simulations, and animations, is also highly valued. But still, SimReal should provide better support to group work, collaboration, discussion, and group dynamics in classroom.

Secondly, at the classroom level, SimReal helps to explore variation and regularities in the way mathematics is taught, e.g., vary a parameter to see the effect of a graph, etc. SimReal fosters differentiation and individualization, and student autonomy so that the students can work at their own pace without much interference from the teacher.

Thirdly, at the mathematics subject level, the majority of the students think that SimReal visualizations are useful to gain mathematical knowledge that is otherwise difficult to acquire. They considered SimReal as a tool that has a high quality of mathematical content. Likewise, the students think that the mathematical notations and symbols of SimReal are correct and sound, which is an expression of high mathematical fidelity. Moreover, SimReal facilitates various activities that engage students in mathematical problem-solving that fosters reflection and mathematical understanding through simulation of mathematical concepts.

Fourthly, in terms of affordances at the assessment levels, the overwhelming majority of the students revealed that SimReal does not provide several types of feedback, question types (e.g. multiple choices), and statistics in terms of scores or grade, even though visualizations and dynamic simulations can be considered as a type of feedback.

Finally, in terms of socio-cultural affordances, the students think that SimReal is appropriate to use in teacher education, mostly at the secondary school as it enables to concretize the curriculum at this level, but in a lesser degree at the primary and middle school level. As a result, many students think that SimReal does not take sufficiently into account the requirement for adapted education.

Affordances Associated with Pythagoras' Theorem

To assess students' experiences with SimReal when engaging with mathematical activities, two specific tasks were given to the students: Pythagoras' and Square theorem. Asking task-based questions provides supplementary, more nuanced and detailed information on affordances and constraints of SimReal.

Twenty-one (21) different examples of Pythagoras' theorem were presented to the students (See Web site in the reference list). To assess students' mathematical engagement with the theorems, seven (7) specific questions were asked.

1.Make a list among 002,…,021 of what you feel is the 5 best of understanding Pythagoras' theorem

Most students agreed that the best methods are 002 and 003 (Fig. 2), followed by the other methods. The students suggested a variety of preferences. The most relevant ones are in order of priority: 2-3-17-13-9; 2-3-11-14-15, 2-3-13-20-21, and 2-3-7-11-17.

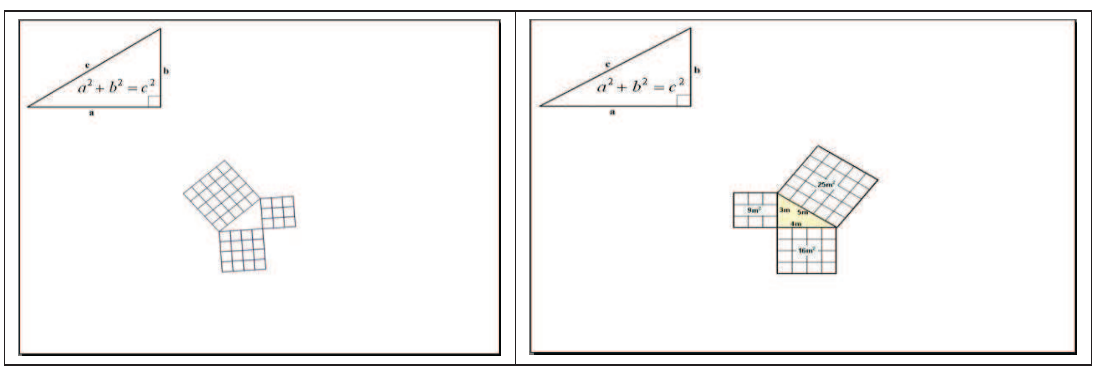

Figure 2. Most preferred methods to engage with Pythagoras' theorem

2.Make a list among 013, …, 016 of what you feel is the to best ways of understanding Pythagoras' Theorem using digital tools

The most preferred methods are 13 and 15 in order of priority (Fig. 3), otherwise the students suggested 15-13, 15-16, 13-14, 15-17, and 19-14.

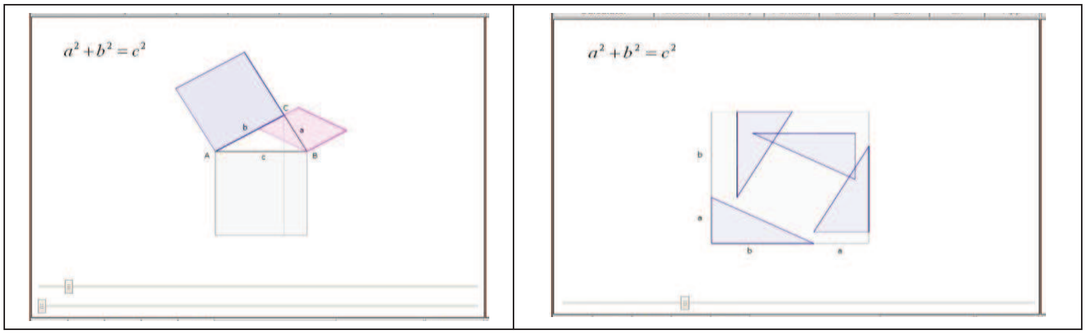

Figure 3. Most preferred methods using digital tools

3. Do you prefer pen-paper proof (002,..,012) or computer proof (013,..,019)?

Both pen-paper and computer methods were considered as acceptable to prove Pythagoras' theorem. Students preferring pen-paper proofs gave different reasons. One argument is that the students can use parts of the proof manually on the blackboard by drawing the mathematical symbols. Another group of students preferred pen-paper proofs, followed by digital proofs. Similarly, some students suggested using pen-paper proofs, and then digital proofs to illustrate mathematical activities done on paper. Finally, another group preferred pen-paper proofs based on examples 001, 002 and 005.

Likewise, students preferring computer proofs gave different reasons. Firstly, a digital proof is more visual and more motivating for them. Secondly, visualizing the figure provides a correct and concrete way of understanding the theorem, and it is easier to change the figure at the same time. Another reason why a computer proof is preferred over pen-paper is that digital tools are more dynamic than pen-paper techniques. Finally, it is easier to move back and forth (or between) the computer simulations, and if done by hand, drawings may be inaccurate.

Finally, one group stated that pen-paper methods are better for younger students, in contrast to older students who can use digital tools. Children can also use computer proofs, but there should not be many disturbing elements in the tool, because they can easily get confused. Clearly, there needs to be more concrete examples and good illustrations.

4.If you should combine one of the 11 pen/paper examples (002,…,012) and one of the 7 simulations (013,..019) which of them would you prefer?

A combination of pen-paper and digital simulations is the most preferred method for all students. Examples of combinations are: 6-15, 12-15, 3-13, 6-16, 2-17, 5-15 (twice), 10-16, and 5-19. One reason for using 5-15 is that 15 combined with 5 provides a dynamic representation that is more intuitive, while 5 shows the algebraic expression that explains the formula for Pythagoras' theorem. Furthermore, the combination of 10 and 16 is a good choice, because colours make it easier to understand the different parts of the theorem and how these are combined into a whole.

5.Do you think that teaching different ways of Pythagoras' theorem by combining pen/paper and simulations would help in the understanding of this topic, or do you think it would be confusing for the pupils/students?

Most students agreed that variation is the best way to teach the theorem for many different reasons. Firstly, students need pen-paper techniques to see the theorem algebraically, in combination with digital visualizations to illustrate the context of use. Secondly, displaying the theorem in different ways will pay off, and it gives students a reinforcing effect. Similarly, using pen-paper methods combined with digital tools would enrich students' understanding of the topic by giving them more representations to hinge their knowledge on. Likewise, it is an advantage to combine algebraic and geometric representations to learn

Pythagoras' theorem, because several forms of representation are meaningful. It is therefore recommended to find out who can benefit from this strategy, and then teach the theorem accordingly. Thus, this strategy may result in two groups of students being taught using two different methods. Another suggestion argues for more geometric explanations both on paper and computer, but without drawing on simulation 16. This would be confusing in a class at the elementary school level. Clearly, different students will learn in different ways. If teachers focus on students lacking conceptual understanding, they will be able to understand how to help them. In contrast, a digital proof may be confusing if it does not show what is worthwhile, and how to foster understanding. Finally, if students use dynamic simulations, then it would be easier to show how the theorem works. On the other hand, while doing mathematics with pen and paper, students can go deeper into the theorem and make it their own in an easier way than the computer proof. This method can be customized for students with difficulties in understanding.

Affordances Associated with the Square Theorem

Six (6) different examples of Square theorem were presented to the students (See web site in the reference list). To assess the students' mathematical engagement with the Square theorem, 5 specific questions were asked.

1. *Make a list of what you feel is the best ways of understanding the three square-theorems*

Several students think it is useful to visualize the algebraic expression of the theorem by combining pen-paper techniques using symbolic calculations and digital simulations, but preferably using the digital simulation first (Fig.4). Alternatively, another student suggested a combination of a geometric explanation, algebraic calculation, and calculation with numbers, and finally, work with related tasks. Similarly, the students recommended to present the figures and then let them describe what they observe. The best way is to show the geometric figures visually, because it is easy to show many symbols at the same time. The students also proposed preferences that work best in order of priority: 003-002-004-006-001, 003-002-004-005-006-001, or 004-005-006 (only three choices in this case).

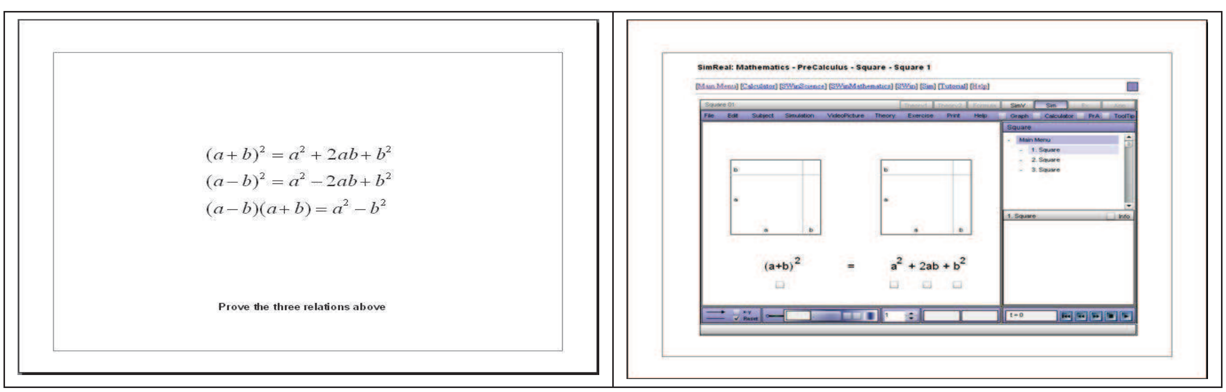

Figure 4. A combination of pen-paper and digital simulation with SimReal

2. Do you prefer paper proof (001, 002, 003) or computer proof (004, 005, 006)?

The majority of the students preferred a combination of pen-paper and computer proofs, because it is easier to understand the theorem if they get it in different ways. Both methods complement each other. Two students preferred starting with pen-paper, and then take it over on the digital tool. In contrast, many preferred getting evidence of the proof using SimReal, which makes it easier to understand it. Students can also watch some videos individually to understand the theorem, but without much discussion with other students. Alternatively, it may be useful to show the computer proof first, since it is visually more intuitive, while pen-paper techniques can be used to illustrate the tasks.

3. In what way do you think the use of digital tool can improve a better understanding of square theorems?

Students believed that digital tools are a good aid in teaching, because they are visually appealing. At the same time, the drawings are much better on the computer than on the board. The visualization aspect is the most important feature when using digital tools as a means for better understanding. More explicitly, digital tools like SimReal make it easier to visualize the theorem, thus showing what happens instead of the formula shown from example 001-003. Other students suggested alternative solutions, e.g., provide an understanding when using algebraic expressions and geometric figures without reference to digital tools, or fostering understanding by guessing the meaning of the symbols/characters that represent lengths in a figure. Finally, two students were unable to decide which method works best. They must share some experiences with other pupils before they can take a decision.

4.*Give some comments about how you could think to improve either by pen/paper or digital tool the understanding of Square theorems*

The students provided several answers to this question. A combination seems to be the best strategy for four students. Increased understanding could be achieved by combining visualizations of the digital tool with the algebraic proofs on paper. Alternatively, one can start with pen and paper to get the basics, and then use digital aids to understand the Square theorem. Another suggestion is to let the students use the resources they have at their disposal, either digital tools or pen-paper techniques or both. Alternatively, if students are able to use digital simulations by themselves, they can do it at their own pace, instead of rewinding back and forth in a video. Colour coding of the algebraic expressions of the figure can also make it easier for students to see how the simulations work. Another reason is that pupils at the primary school level do not have to undergo the Square theorem. However, if they might go through it, they would use a lot of visual images and videos, because the curriculum is presented in a concrete and visual way, which allows them to understand what actually happens and what they can expect.

5.Do you prefer learning the Square theorem in one way or do you feel a better understanding when learning it in different ways?

Most students agree that different learning methods are better than only one unique way. This is clearly expressed in their comments.

> *It is an advantage to use several approaches, but it is important to take one at a time, otherwise students can get confused. It is important to check if the student has understood, instead of showing several alternative examples. In contrast, students can explore alternative proofs.*
>
> *A better understanding can be achieved by learning the theorem using different methods since different students learn in different ways.*
>
> *Several perspectives can provide a deeper understanding of the theorem.*
>
> *Quadratic numbers should be used in many contexts and preferably in solving problems.*
>
> *If students had to learn about the theorem by themselves, it would be an advantage to combine different methods, but it is recommended to use digital simulations before the algebraic expression on pen and paper.*

Learning the Square theorem in several ways would be most beneficial. It is better to learn the theorem in different ways because one can see through it from different angles, and consequently since students learn in different ways, one cannot understand one without the other.

Discussion

The first issue raised in this study is the identification of affordances at four different levels: technological, pedagogical, mathematical, and socio-cultural level. These four broad categories of affordances provide insight into the potentialities and constraints of SimReal in educational settings.

In terms of technological affordances, the students were globally satisfied with SimReal in terms of availability of mathematical content, and open accessibility of the tool. However, SimReal still does not have an intuitive user interface or attractive design, and management facilities, e.g. a user manual, which could negatively affect novice students without experience with visualization tools. Despite this limitation, the students used SimReal to simulate various mathematical tasks, including Pythagoras' and Square theorem to achieve the didactical goal of learning mathematics. SimReal allowed them to explore a wide range of dynamic visualizations of the theorems and gave them the opportunity to create connections between symbolic and visual representations of the theorems. In addition, the tool is highly congruent with paper-pencil techniques as the exercises with Pythagoras' and Square theorems clearly show.

In terms of pedagogical affordances, several issues have been addressed: Motivation, student autonomy, individualization, differentiation, variation, and activities. Students evaluated these affordances of SimReal positively as they provide action possibilities. Moreover, SimReal is useful because it combines various activities (problem-solving, video lectures, live streaming, and simulation), and several ways of representing mathematical knowledge (texts, graphs, symbols, animations, visualizations). Enhanced motivation is achieved through realistic mathematical tasks, dynamic simulations and visualizations, and a combination of these. Moreover, most students think that the tool is adapted to their age and development level, which is a motivational factor in keeping them engaged in mathematics. Beyond motivational issues, many students think that SimReal enables a high degree of student autonomy allowing them to work at their own pace.

Likewise, in terms of mathematical affordances at the pedagogical level, SimReal provides a high quality of mathematical content, e.g., mathematical notations and symbols that are correct and sound. the results also show that doing mathematics with paper and pencil it is still important to stimulate learning when doing mathematics with paper and pencil techniques. Moreover, the results indicate that SimReal and the digital simulations of Pythagoras and Square theorems can be used as an alternative to achieve variation in teaching. The tasks show that SimReal facilitates various activities with the theorems, and it can be used in combination with pen-paper proofs. This is in line with the research literature that indicates that variation in teaching is important because students learn in different ways (Hadjerrouit, 2017). This is an important pedagogical affordance when doing mathematics depending on the students' prerequisite and the mathematical task being solved.

In terms of specific affordances associated with task-based tasks, the results achieved so far confirm the affordances at the student level, in particular in terms of variation and combination of different types of mathematical representations.

In terms of affordances at the assessment level, SimReal still does not provide several types of feedback, question types, and statistics in form of scores or grade.

Finally, in terms of socio-cultural affordances, SimReal can be used in teacher education, mostly at the secondary school, but in a lesser degree at the primary and middle school level.

Conclusion and Future Work

This study aims to critically evaluate the affordances of SimReal by means of quantitative and qualitative methods based on the concept of affordance. The data collected by means of survey questionnaires with open-ended questions and task-based data collection methods provide an important amount of information that can be used to improve the teaching and learning of mathematics using SimReal. The results achieved so far are promising, and these can be used to foster mathematical understanding beyond

Pythagoras' and Square theorem. Future research work will focus partly on programming issues and other motivating tasks that may enhance mathematical learning in teacher education. Still, SimReal needs to be improved. In terms of technological affordances, there is a need for an intuitive user interface for different types of users as this may play a critical role in the appropriation of the tool. In terms of pedagogical affordances, there is a need for better feedback and review modes, more differentiation and adapted education. Finally, the concept of affordances will be refined to better suit the next cycle of experimentation with SimReal. Finally, data collection and analysis methods will be improved to ensure more validity and reliability. Interviews with students will add more evidence to the data collected so far.

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